Conditional Probability

P(B | A) = **The probability of event B given that A has already occurred.**

Example 1: A bag contains 10 beads—2 black, 3 white, and 5 red. A bead is selected at random. Find the probability of …

a) selecting a white bead, replacing it, and then selecting a red bead.

independent

$P(W) \cdot P(R)$

$\frac{3}{10} \cdot \frac{5}{10} = \frac{15}{100} = \boxed{\frac{3}{20}}$

b) selecting a white bead, not replacing it, and then selecting a red bead.

dependent

$P(W) \cdot P(R \mid W)$

$\frac{3}{10} \cdot \frac{5}{9} = \frac{15}{90} = \boxed{\frac{1}{6}}$

c) selecting 3 non-red beads without replacement.

dependent

$\frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} = \boxed{\frac{1}{12}}$

Are the above events independent or dependent?

Example 2: In a sales effectiveness seminar, a group of sales representatives tried two approaches to selling a customer a new automobile: the aggressive approach and the passive approach. From 1160 customers, the following record was kept:

	Sale	No Sale	Row Total
Aggressive	270	310	580
Passive	416	164	580
Column Total	686	474	1160

Suppose that a customer is selected at random from the 1160 participating customers. Compute the following:

1. P(sale) $\frac{686}{1160} = \frac{343}{580}$

2. P(sale | aggressive approach) $\frac{270}{580} = \frac{27}{58}$

	Sale	No Sale	Row Total
Aggressive	270	310	580
Passive	416	164	580
Column Total	686	474	1160

3. P(sale | passive approach) $\frac{416}{580} = \frac{104}{145}$

4. P(aggressive and sale) $\cap$ $\frac{270}{1160} = \frac{27}{116}$

5. P(passive and sale) $\cap$ $\frac{416}{1160} = \frac{52}{145}$

6. P(no sale) $\frac{474}{1160} = \frac{237}{580}$

	Sale	No Sale	Row Total
Aggressive	270	310	580
Passive	416	164	580
Column Total	686	474	1160

7. P(no sale | aggressive) $\frac{310}{580} = \frac{31}{58}$

8. P(aggressive or sale)

overlapping

$$P(A) + P(S) - P(A \cap S)$$
$$\frac{580}{1160} + \frac{686}{1160} - \frac{270}{1160}$$
$$= \frac{996}{1160} = \frac{249}{290}$$