DOT PRODUCT WORKSHEET

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For the following problems find the dot product of the two vectors:

1. $\vec{a} =< 3, 5 >$ and $\vec{b} =< -2, 3 >$

$\vec{a} \cdot \vec{b} = -6 + 15 = \boxed{9}$

2. $\vec{c} =< 1, -7 >$ and $\vec{d} =< -2, -4 >$

$\vec{c} \cdot \vec{d} = -2 + 28 = \boxed{26}$

3. $\vec{e} = \frac{2}{3}\vec{\imath} + \frac{3}{2}\vec{\imath}$ and $\vec{f} = -\frac{5}{2}\vec{\imath} + \frac{4}{3}\vec{\jmath}$

$\vec{e} \cdot \vec{f} = -\frac{10}{6} + \frac{12}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$

4. $\vec{g} = -3\vec{\imath} + 5\vec{\jmath}$ and $\vec{h} = -5\vec{\imath} - 3\vec{\jmath}$

$\vec{g} \cdot \vec{h} = 15 + (-15) = \boxed{0}$

* $\vec{g}$ and $\vec{h}$ are orthogonal!

For the following problems find the angle between the two given vectors:

5. $\vec{u} = 3\vec{\imath} - 5\vec{\jmath}$ and $\vec{v} = -6\vec{\imath} - 2\vec{\jmath}$

$\cos\theta = \frac{-18 + 10}{\sqrt{34} \cdot \sqrt{40}}$

$\theta = \cos^{-1}\left(\frac{-8}{\sqrt{1360}}\right) = \boxed{102.53^\circ}$

6. $\vec{v} =< -8, -3 >$ and $\vec{w} =< 3, -8 >$

$\cos\theta = \frac{-24 + 24}{\sqrt{73} \cdot \sqrt{73}}$

$\theta = \cos^{-1}(0) = 90^\circ$

7. $\vec{u} = \vec{\imath} + 3\vec{\jmath}$ and $\vec{v} = -2\vec{\jmath}$

$\cos\theta = \frac{0 - 6}{\sqrt{10} \cdot \sqrt{4}}$

$\theta = \cos^{-1}\left(\frac{-6}{\sqrt{40}}\right) = \boxed{161.57^\circ}$

8. $\vec{v} = \frac{2}{3}\vec{\imath} + \frac{3}{2}\vec{\imath}$ and $\vec{w} = -\frac{5}{2}\vec{\imath} + \frac{4}{3}\vec{\jmath}$

$\cos\theta = \frac{-\frac{10}{6} + \frac{12}{6}}{\sqrt{\frac{4}{9} + \frac{9}{4}} \cdot \sqrt{\frac{25}{4} + \frac{16}{9}}} = \frac{\frac{2}{6}}{\sqrt{\frac{97}{36}} \cdot \sqrt{\frac{301}{36}}}$

$\theta = \cos^{-1}\left(\frac{1}{3} \Big/ \sqrt{\frac{29197}{1296}}\right) = \boxed{85.97^\circ}$

For the following problems determine if the vectors are orthogonal (perpendicular):

9. $\vec{v} =< -8, -3 >$ and $\vec{w} =< 3, -8 >$

$\vec{v} \cdot \vec{w} = -24 + 24 = 0$

orthogonal!

10. $\vec{v} =< 0, -7 >$ and $\vec{w} =< 11, -2 >$

$\vec{v} \cdot \vec{w} = 0 + 14 = 14$

not!

11. $\vec{u} = \vec{\imath} + 2\vec{\jmath}$ and $\vec{v} = 2\vec{\imath} - \vec{\jmath}$

$\vec{u} \cdot \vec{v} = 2 - 2 = 0$

orthogonal!

12. $\vec{u} = 10\vec{\imath} - 2\vec{\jmath}$ and $\vec{v} = 2\vec{\imath} + 9\vec{\jmath}$

$\vec{u} \cdot \vec{v} = 20 - 18 = 2$

not!

For the following problems find the dot product of the vectors given their magnitude and the angle in between the two vectors:

$\vec{u} \cdot \vec{v} = ||\vec{u}|| \cdot ||\vec{v}|| \cdot \cos\theta$

13. $If\ ||\vec{a}|| = 7\,, ||\vec{b}|| = 8,\ and\ \theta = 155°$

$\vec{a} \cdot \vec{b} = 7 \cdot 8 \cdot \cos 155° = \boxed{-50.75}$

14. $If\ ||\vec{c}|| = 3\,,\ ||\vec{d}|| = 11,\ and\ \theta = 65°$

$\vec{c} \cdot \vec{d} = 3 \cdot 11 \cdot \cos 65° = \boxed{13.95}$

15. $If\ ||\vec{e}|| = 5\,,\ ||\vec{f}|| = 7,\ and\ \theta = 102°$

$\vec{e} \cdot \vec{f} = 5 \cdot 7 \cdot \cos 102° = \boxed{-7.28}$

16. $If\ ||\vec{g}|| = 11\,,\ ||\vec{h}|| = 2,\ and\ \theta = 14°$

$\vec{g} \cdot \vec{h} = 11 \cdot 2 \cdot \cos 14° = \boxed{21.35}$

For the following problems find the angle between the two vectors given their dot product:

17. $If\ ||\vec{g}|| = 10\,,\ ||\vec{h}|| = 20,\ and\ \vec{g} \cdot \vec{h} = -35$ $\quad find\ \theta$

$\cos\theta = \dfrac{-35}{10 \cdot 20}$

$\theta = \cos^{-1}\left(\dfrac{-35}{200}\right) = \boxed{100.08°}$

18. $If\ ||\vec{v}|| = 12\,,\ ||\vec{w}|| = 6,\ and\ \vec{v} \cdot \vec{w} = 67$ $\quad find\ \theta$

$\cos\theta = \dfrac{67}{12 \cdot 6}$

$\theta = \cos^{-1}\left(\dfrac{67}{72}\right) = \boxed{21.48°}$