Double Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
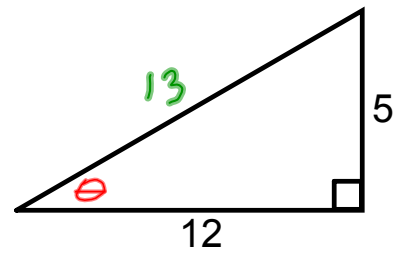$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Example 1:

$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{5}{12}$

13, 5, 12

Find:

1) $\sin 2\theta = 2\cdot\sin\theta\cdot\cos\theta$
$$= 2\left(\frac{5}{13}\right)\cdot\left(\frac{12}{13}\right) = \boxed{\frac{120}{169}}$$
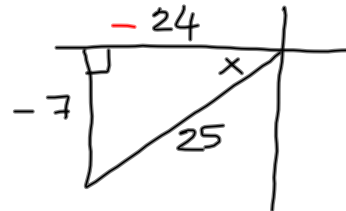
2) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$$

3) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{5}{12}\right)}{1-\left(\frac{5}{12}\right)^2}$
$$= \frac{\frac{5}{6}}{\frac{144}{144}-\frac{25}{144}} = \frac{\frac{5}{6}}{\frac{119}{144}} = \frac{5}{6}\cdot\frac{144}{119} = \boxed{\frac{120}{119}}$$

Example 2:

Given $\sin x = -\dfrac{7}{25}$ when $\pi < x < \dfrac{3\pi}{2}$

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-24, x, -7, 25

Find:

1) $\sin 2\theta = 2\cdot\sin\theta\cdot\cos\theta$
$$= 2\left(\frac{-7}{25}\right)\left(\frac{-24}{25}\right) = \boxed{\frac{336}{625}}$$

2) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$$= \left(\frac{-24}{25}\right)^2 - \left(\frac{-7}{25}\right)^2 = \frac{576}{625} - \frac{49}{625} = \boxed{\frac{527}{625}}$$

3) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{7}{24}\right)}{1-\left(\frac{7}{24}\right)^2} = \frac{\frac{7}{12}}{\frac{576}{576}-\frac{49}{576}} = \frac{\frac{7}{12}}{\frac{527}{576}}$
$$= \frac{7}{12}\cdot\frac{576}{527} = \boxed{\frac{336}{527}}$$

$\frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{336}{625}}{\frac{527}{625}} = \boxed{\frac{336}{527}}$