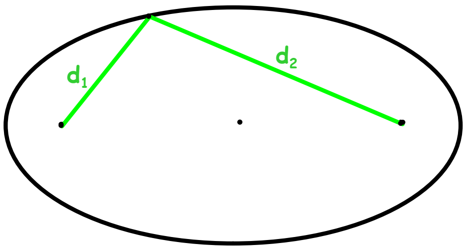
Ellipses – Graphing

Ellipse

An ellipse is the set of all points (x, y) the sum of whose distances from two distinct fixed points (foci) is constant.

The foci of an ellipse lie on the major axis, *c* units from the center where $c^2 = a^2 - b^2$.



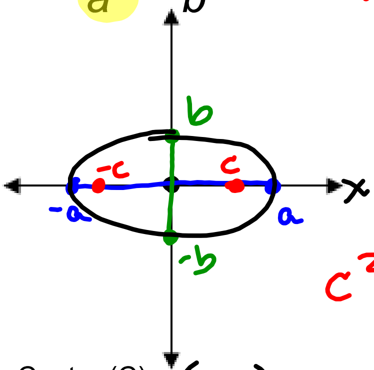
$d_1 + d_2 = \text{constant}$

click blue dot below for interactive demo:
http://mathworld.wolfram.com/Ellipse.html

Horizontal Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 > b^2$
$a > b$

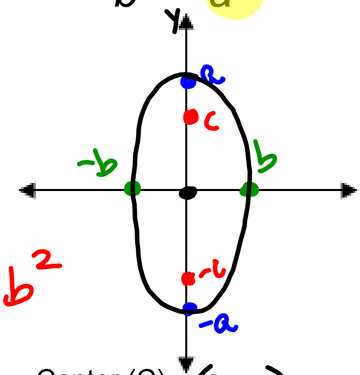


$c^2 = a^2 - b^2$

Center (C) $(0,0)$
Vertices (V) $(-a,0)$ $(a,0)$
Co-vertices (CV) $(0,-b)$ $(0,b)$
Foci (F) $(-c,0)$ $(c,0)$
major axis length = $2a$
minor axis length = $2b$

Vertical Ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



$(\pm b, 0)$

Center (C) $(0,0)$
Vertices (V) $(0,-a)$ $(0,a)$
Co-vertices (CV) $(-b,0)$ $(b,0)$
Foci (F) $(0,-c)$ $(0,c)$
major axis length = $2a$
minor axis length = $2b$

Ellipses - Translated/Shifted Ellipses

Horizontal Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Vertical Ellipse

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Center: (h,k)

"b" → distance from center to co-vertices

"a" → distance from center to vertices

$c^2=a^2-b^2$

"c" → distance from center to foci

Example 1: horizontal

$$\frac{x^2}{25}+\frac{y^2}{4}=1$$

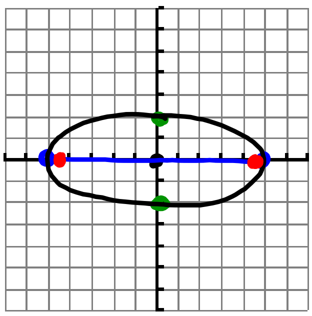
$a=\pm 5$

$b=\pm 2$

$c^2=25-4$

$c^2=21$

$c=\pm\sqrt{21}$



C (0, 0)

V (−5, 0) (5, 0) (±5, 0)

CV (0, −2) (0, 2)

F $(0+\sqrt{21}, 0)$ $(0-\sqrt{21}, 0)$

major length = 10

minor length = 4

Example 2: vertical

$$\frac{x^2}{1}+\frac{y^2}{9}=1$$

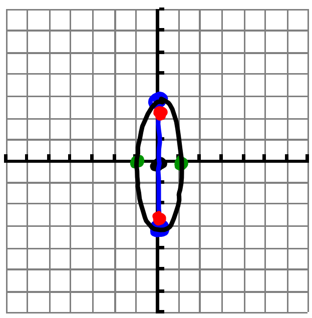
$a=3$

$b=1$

$c^2=8$

$c=\sqrt{8}$

$c=2\sqrt{2}$



C (0, 0)

V (0, −3) (0, 3)

CV (−1, 0) (1, 0)

F $(0, 0\pm 2\sqrt{2})$

major length = 6

minor length = 2

Example 3:

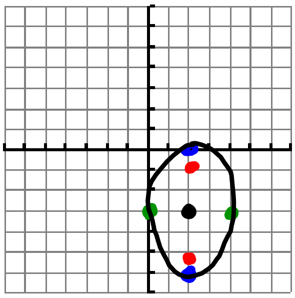
$$\frac{(x-2)^2}{4}+\frac{(y+3)^2}{9}=1$$

$a = 3$

$b = 2$

$c^2 = 5$

$c = \pm\sqrt{5}$



C $(2,-3)$

V $(2,0)$ $(2,-6)$

CV $(0,-3)$ $(4,-3)$

F $(2,-3\pm\sqrt{5})$

major length = 6

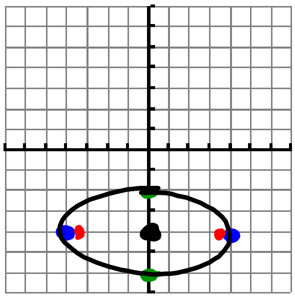
minor length = 4

Example 4:

$$\frac{x^2}{16}+\frac{(y+4)^2}{4}=1$$

$c^2 = 12$

$c = 2\sqrt{3}$



C $(0,-4)$

V $(-4,-4)$ $(4,-4)$

CV $(0,-2)$ $(0,-6)$

F $(0\pm2\sqrt{3},-4)$

major length = 8

minor length = 4