Factors, Zeros, and Roots: Oh My! - Intro to Solving Polynomial Equations

Part I: Solving Quadratic Equations

Solving polynomials that have a degree greater than those solved in Coordinate Algebra and Analytic Geometry is going to require the use of skills that were developed when we solved quadratics last year. Let's begin by taking a look at some second degree polynomials and the strategies used to solve them. These equations have the form $ax^2 + bx + c = 0$, and when they are graphed the result is a parabola.

1. **Factoring** is used to solve quadratics of the form $ax^2 + bx + c = 0$ when the roots are rational. Find the roots of the following quadratic functions by factoring:

a. $f(x) = x^2 - 5x - 14$

b. $f(x) = x^2 - 64$

c. $f(x) = 6x^2 + 7x - 3$

2. When roots are NOT rational, quadratic equations of the form $x^2 + bx + c = 0$, where b is even, can be easily solved by **completing the square**. Find the roots of the following quadratic equations by completing the square:

a. $f(x) = x^2 - 6x + 4$

b. $f(x) = x^2 + 8x + 20$

c. $f(x) = x^2 + 2x - 17$

3. Another option for solving any quadratic equation is to use the **quadratic formula**. Remember, a quadratic equation written in $ax^2 + bx + c = 0$ has solution(s) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Also remember that $b^2 - 4ac$ is the discriminant and gives us the ability to determine the nature of the roots.

$$b^2 - 4ac \begin{cases} > 0 & 2 \quad \textit{real roots} \,(\textit{rational or irrational}) \\ = 0 & 1 \quad \textit{real root} \qquad (\textit{rational}) \\ < 0 & 0 \quad \textit{real roots} \quad (2\ \textit{imaginary}) \end{cases}$$

Find the roots for each of the following using the quadratic formula. Also, describe the number and nature of these roots.

a. $f(x) = 4x^2 - 2x + 9$

b. $f(x) = 3x^2 + 4x - 8$

c. $f(x) = x^2 - 5x + 9$

Now, we can start to solve polynomial equations of higher degree using a combination of synthetic division and solving quadratic equation techniques! Let's investigate …