Half-Angle Identities

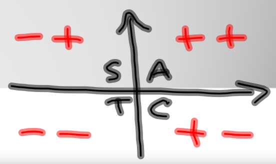
$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

Your answer will NOT be both $\pm$. The signs of sin(u/2) and cos(u/2) depend on the quadrant in which angle (u/2) lies.

based on "A.S.T.C." rule

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$



Ex. 1: Use the half-angle formula to find the exact values of …

a) $\sin 105^\circ = \sin\left(\frac{210^\circ}{2}\right) = +\sqrt{\frac{1-\cos 210^\circ}{2}} = \sqrt{\frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{2}}$

$\downarrow$ QII

$+$ in

$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

b) $\cos 105^\circ = \sin\left(\frac{210^\circ}{2}\right) = -\sqrt{\frac{1+\cos 210^\circ}{2}} = -\sqrt{\frac{1+\left(\frac{-\sqrt{3}}{2}\right)}{\frac{2}{1}}}$

$\downarrow$ QII

$-$ in

$= -\sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = -\frac{\sqrt{2-\sqrt{3}}}{\sqrt{4}} = \boxed{-\frac{\sqrt{2-\sqrt{3}}}{2}}$

c) $\tan 105^\circ = \frac{1-\cos 210^\circ}{\sin 210^\circ} = \frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{-\frac{1}{2}}$

$\tan\left(\frac{210^\circ}{2}\right)$

$= \frac{2+\sqrt{3}}{\cancel{2}}\cdot\left(-\frac{\cancel{2}}{1}\right) = \boxed{-2-\sqrt{3}}$

Ex. 2: Use the half-angle formula to find the exact values of …

a) $\sin\frac{\pi}{8} = \sin\left(\frac{\pi/4}{2}\right) = +\sqrt{\frac{1-\cos\pi/4}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}} = \boxed{\frac{+\sqrt{2-\sqrt{2}}}{2}}$

$+$ QI

b) $\cos\frac{\pi}{8} = \cos\left(\frac{\pi/4}{2}\right) = +\sqrt{\frac{1+\cos\pi/4}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \boxed{\frac{+\sqrt{2+\sqrt{2}}}{2}}$

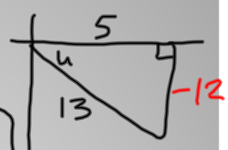
c) $\tan\frac{\pi}{8} = \tan\left(\frac{\pi/4}{2}\right) = \frac{1-\cos\pi/4}{\sin\pi/4} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{2}}$

$= \frac{(2-\sqrt{2})}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}-2}{2}$

$= \boxed{\sqrt{2}-1}$

Ex. 3: Find the exact values of sin(u/2), cos(u/2) and tan(u/2) using the half-angle formulas.

$$\cos u = \frac{5}{13},\quad \frac{3\pi}{2} < u < 2\pi$$



$u \to Q4$

$\frac{u}{2} \to Q\frac{4}{2}$

$\sin\frac{u}{2} = +\sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\frac{5}{13}}{2}} = \sqrt{\frac{13-5}{13}\cdot\frac{1}{2}} = \sqrt{\frac{4}{13}}$

$= \frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{2\sqrt{13}}{13}}$

$\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\frac{5}{13}}{2}} = -\sqrt{\frac{\frac{13+5}{13}}{\frac{2}{1}}} = -\sqrt{\frac{18}{13}\cdot\frac{1}{2}} = -\sqrt{\frac{9}{13}}$

$= -\frac{3}{\sqrt{13}} = \boxed{\frac{-3\sqrt{13}}{13}}$

$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{1-\frac{5}{13}}{-\frac{12}{13}} = \frac{\frac{13-5}{13}}{\frac{-12}{13}} = \frac{8}{13}\cdot\left(-\frac{13}{12}\right) = \boxed{-\frac{2}{3}}$