

Identity and Inverse Matrices

Square

$n \times n$ identity matrix - the matrix that has 1's on the main diagonal and 0's elsewhere.

$$I_{2 \times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- ★ If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $IA=A$ and $AI=A$.
- ★ If B is any $m \times n$ matrix, then $I_{m \times m}B=B$ and $BI_{n \times n} = B$.

The Inverse of a 2x2 Matrix:

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

$$\text{then } A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{ provided } |A| \neq 0.$$

determinant
of A

switch
entries

take
opposite
sign

Ex 1. Find each inverse:

$$\text{a. } A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$|A| = 6 - 4 = 2$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}$$

$$\text{b. } A = \begin{bmatrix} 6 & 1 \\ -8 & -2 \end{bmatrix}$$

$$|A| = -12 - (-8) = -4$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} -2 & -1 \\ 8 & 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ -2 & -\frac{3}{2} \end{bmatrix}$$