

SOLVING NON-RIGHT TRIANGLES



YOU SHOULD KNOW...

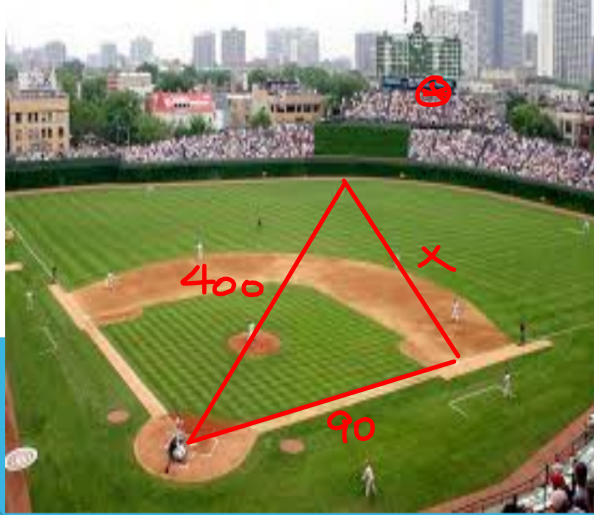
Solving a triangle means finding all sides and angles.

The Laws of Sines and Cosines work for all triangles – not just right triangles.



BASEBALL APPLICATION

During a baseball game an outfielder caught a ball hit to dead center field, 400 feet from home plate. If the distance from home plate to first base is 90 feet, how far does the outfielder have to throw the ball to get it to first base?

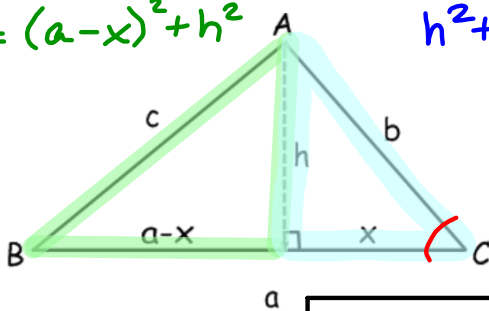


THE LAW OF COSINES

USE FOR SAS OR SSS

$$c^2 = (a-x)^2 + h^2$$

$$h^2 + x^2 = b^2$$



DERIVE
THE LAW
OF COSINES

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b \cdot \cos C = \frac{x}{b}$$

$$x = b \cos C$$

$$c^2 = (a-x)^2 + h^2$$

$$c^2 = a^2 - 2ax + x^2 + h^2$$

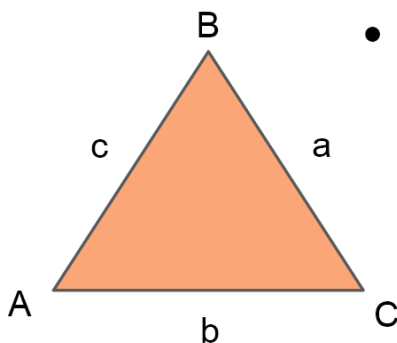
$$c^2 = a^2 - 2ax + b^2$$

$$c^2 = a^2 + b^2 - 2ax$$

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos C$$

LAW OF COSINES

- $a^2 = b^2 + c^2 - 2bc\cos A$
- $b^2 = a^2 + c^2 - 2ac\cos B$
- $c^2 = a^2 + b^2 - 2ab\cos C$



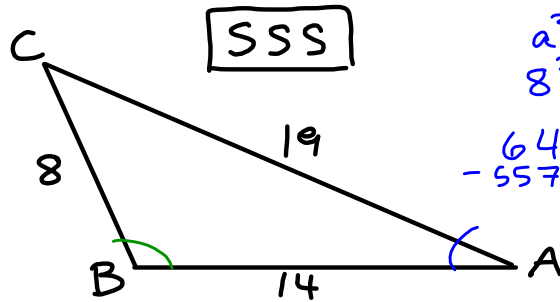
Δ FUN

$$f^2 = u^2 + n^2 - 2un \cdot \cos F$$



EX. 1: FIND ALL THE ANGLES.

$a = 8$ ft, $b = 19$ ft, and $c = 14$ ft



$$\begin{aligned}a^2 &= b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A \\8^2 &= 19^2 + 14^2 - 2(19)(14)\cos A \\64 &= 557 - 532 \cdot \cos A \\-557 & \quad -557 \\-493 &= -532 \cos A \\-532 & \quad -532 \\ \frac{493}{532} &= \cos A\end{aligned}$$

$$A = \cos^{-1}\left(\frac{493}{532}\right) = \boxed{22.1^\circ}$$

$$\begin{aligned}b^2 &= a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos B \\19^2 &= 8^2 + 14^2 - 2(8)(14)\cos B \\361 &= 260 - 224 \cos B\end{aligned}$$

$$-\frac{101}{224} = \cos B$$

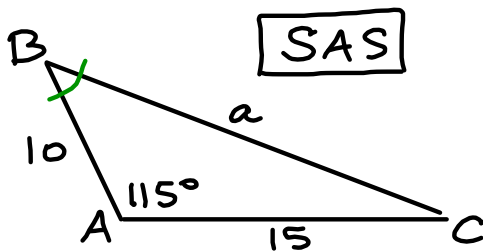
$$B = \cos^{-1}\left(-\frac{101}{224}\right) = \boxed{116.8^\circ}$$

$$C = 180^\circ - 22.1^\circ - 116.8^\circ$$

$$\boxed{C = 41.1^\circ}$$

EX. 2: FIND THE MISSING MEASURES.

$A = 115^\circ$, $b = 15$ cm, and $c = 10$ cm



$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cdot \cos A \\a^2 &= 15^2 + 10^2 - 2(15)(10)\cos 115^\circ \\a^2 &= 451.7854785 \dots\end{aligned}$$

$\sqrt{\text{ANS}}$

$$\boxed{a = 21.3}$$

$$\begin{aligned}b^2 &= a^2 + c^2 - 2ac \cdot \cos B \\15^2 &= 21.3^2 + 10^2 - 2(21.3)(10)\cos B \\225 &= 553.69 - 426 \cos B\end{aligned}$$

$$-\frac{328.69}{426} = \cos B$$

$$B = \cos^{-1}\left(\frac{328.69}{426}\right) = \boxed{39.5^\circ}$$

$$C = 180^\circ - 115^\circ - 39.5^\circ$$

$$\boxed{C = 25.5^\circ}$$