

Matching WS / Activity

$$\begin{aligned} 1. \quad \frac{1}{\cot \theta} + \frac{1}{\tan \theta} &= \tan \theta + \cot \theta \quad \left. \vphantom{\frac{1}{\cot \theta} + \frac{1}{\tan \theta}} \right\} \text{reciprocal} \\ &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \quad \left. \vphantom{\frac{1}{\cot \theta} + \frac{1}{\tan \theta}} \right\} \text{quotient} \\ &= \frac{\sin \theta \cdot \sin \theta + \cos \theta \cdot \cos \theta}{\sin \theta \cdot \cos \theta} \quad \left. \vphantom{\frac{1}{\cot \theta} + \frac{1}{\tan \theta}} \right\} \begin{array}{l} \text{common} \\ \text{denom.} \end{array} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cdot \cos \theta} \quad \left. \vphantom{\frac{1}{\cot \theta} + \frac{1}{\tan \theta}} \right\} \begin{array}{l} \text{add} \\ \text{fractions} \end{array} \\ &= \frac{1}{\sin \theta \cdot \cos \theta} \quad \left. \vphantom{\frac{1}{\cot \theta} + \frac{1}{\tan \theta}} \right\} \text{Pythagorean} \\ &= \boxed{\csc \theta \cdot \sec \theta} \quad \left. \vphantom{\frac{1}{\cot \theta} + \frac{1}{\tan \theta}} \right\} \text{reciprocal} \end{aligned}$$

$$\begin{aligned} 2. \quad \frac{\csc \theta}{\tan \theta + \cot \theta} &= \frac{1}{\frac{\sin \theta}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}}} \quad \left. \vphantom{\frac{\csc \theta}{\tan \theta + \cot \theta}} \right\} \begin{array}{l} \text{change all to} \\ \sin \theta \text{ ? } \cos \theta \end{array} \\ &= \frac{1}{\frac{\sin \theta}{\frac{\sin^2 \theta}{\sin \theta \cdot \cos \theta} + \frac{\cos^2 \theta}{\sin \theta}} \quad \left. \vphantom{\frac{\csc \theta}{\tan \theta + \cot \theta}} \right\} \begin{array}{l} \text{common denom} \\ \text{(same as in \#1)} \end{array} \\ &= \frac{1}{\frac{\sin \theta}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cdot \cos \theta}}} \quad \left. \vphantom{\frac{\csc \theta}{\tan \theta + \cot \theta}} \right\} \text{add fractions} \\ &= \frac{1}{\frac{\sin \theta}{1}} \quad \left. \vphantom{\frac{\csc \theta}{\tan \theta + \cot \theta}} \right\} \text{Pythagorean} \\ &= \frac{1}{\sin \theta} \cdot \sin \theta \cdot \cos \theta \quad \left. \vphantom{\frac{\csc \theta}{\tan \theta + \cot \theta}} \right\} \text{mult. by reciprocal} \\ &= \boxed{\cos \theta} \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{\sin^2 \theta}{\sec^2 \theta - 1} &= \frac{\sin^2 \theta}{\tan^2 \theta} \rightarrow \text{Pythagorean} \\
 &= \frac{\sin^2 \theta}{\frac{\sin^2 \theta}{\cos^2 \theta}} \quad \left. \vphantom{\frac{\sin^2 \theta}{\frac{\sin^2 \theta}{\cos^2 \theta}}} \right\} \text{quotient} \\
 &= \sin^2 \theta \cdot \frac{\cos^2 \theta}{\sin^2 \theta} \quad \left. \vphantom{\sin^2 \theta \cdot \frac{\cos^2 \theta}{\sin^2 \theta}} \right\} \text{mult. by reciprocal} \\
 &= \boxed{\cos^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \cos^2 \theta - \sin^2 \theta \cdot \cos^2 \theta &= \cos^2 \theta (1 - \sin^2 \theta) \rightarrow \text{GCF} \\
 &= \cos^2 \theta (\cos^2 \theta) \rightarrow \text{Pythagorean} \\
 &= \boxed{\cos^4 \theta}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \frac{\tan \theta}{\csc \theta} + \frac{\sin \theta}{\tan \theta} &= \frac{\sin \theta}{\cos \theta} + \frac{\sin \theta}{\frac{1}{\sin \theta}} \quad \left. \vphantom{\frac{\sin \theta}{\frac{1}{\sin \theta}}} \right\} \begin{array}{l} \text{quotient} \\ \text{? reciprocal} \end{array} \\
 &= \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{1} + \frac{\sin \theta}{1} \cdot \frac{\cos \theta}{\sin \theta} \quad \left. \vphantom{\frac{\sin \theta}{1} \cdot \frac{\cos \theta}{\sin \theta}} \right\} \begin{array}{l} \text{mult. by} \\ \text{reciprocal} \end{array} \\
 &= \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos \theta \cdot \cos \theta}{\cos \theta} \quad \left. \vphantom{\frac{\cos \theta \cdot \cos \theta}{\cos \theta}} \right\} \text{common} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \quad \left. \vphantom{\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta}} \right\} \text{add fractions} \\
 &= \frac{1}{\cos \theta} \rightarrow \text{Pythagorean} \\
 &= \boxed{\sec \theta} \rightarrow \text{reciprocal}
 \end{aligned}$$

$$6. \frac{1 - \csc \theta}{\csc \theta} = \frac{1}{\csc \theta} - \frac{\csc \theta}{\csc \theta} \quad \left. \vphantom{\frac{1 - \csc \theta}{\csc \theta}} \right\} \text{separate fraction}$$

$$= \boxed{\sin \theta - 1} \rightarrow \text{reciprocal}$$

$$7. \sin^2 \theta \cot^2 \theta + \sin^2 \theta = \sin^2 \theta (\cot^2 \theta + 1) \rightarrow \text{GCF}$$

$$= \sin^2 \theta (\csc^2 \theta) \rightarrow \text{Pythagorean}$$

$$= \sin^2 \theta \cdot \frac{1}{\sin^2 \theta} \quad \left. \vphantom{\sin^2 \theta \cdot \frac{1}{\sin^2 \theta}} \right\} \text{reciprocal}$$

$$= \frac{\sin^2 \theta}{\sin^2 \theta} = \boxed{1}$$

$$8. \frac{\cos^2 \theta}{(1 - \sin \theta)} \cdot \frac{(1 + \sin \theta)}{(1 + \sin \theta)} \quad \left. \vphantom{\frac{\cos^2 \theta}{(1 - \sin \theta)} \cdot \frac{(1 + \sin \theta)}{(1 + \sin \theta)}} \right\} \text{multiply by "conjugate"}$$

$$= \frac{\cos^2 \theta (1 + \sin \theta)}{1 - \sin^2 \theta}$$

$$= \frac{\cos^2 \theta (1 + \sin \theta)}{\cos^2 \theta} \rightarrow \text{Pythagorean}$$

$$= \boxed{1 + \sin \theta}$$

