PreCalculus Name _____

Review for Midterm Exam

In order to receive bonus points, this worksheet must be completed on **SEPARATE** paper. You must show **ALL** of your **WORK** and have it **ORGANIZED** neatly. This must be turned in on the day of your midterm exam.

Conics

1. Graph and provide the requested information:
   Circles: Center, Radius
   Ellipses: Center, Vertices, Co-vertices, foci, major and minor axis length
   Hyperbolas: Center, Vertices, Foci, and Asymptotes
   Parabolas: Vertex, Focus, Directrix, End Points of Latus Rectum

a. $(x+1)^2+(y-3)^2=10$ b. $\frac{(x-2)^2}{9}+\frac{y^2}{25}=1$ c. $16x^2-9y^2=144$

d. $\frac{(y-2)^2}{25}-\frac{(x+3)^2}{4}=1$ e. $(x+4)+(y-2)^2=0$ f. $4(y-1)^2=16(x-5)$

2. Name the conic and write it in standard form:

   a. $x^2 + y^2 - 6x - 2y + 1 = 0$

   b. $6x^2-12=6y^2$

   c. $9x^2 + 4y^2 + 54x - 16y + 61 = 0$

   d. $9x^2-4y^2+36x-8y-40=0$

   e. $x^2+x-y=5$

3. Write the standard form of the given conic using the given information:

   a. circle with center (–2, 3) and diameter 8

   b. horizontal ellipse with center at (3, –4); major axis length 8; minor axis length 4

   c. circle with center (1, 4) and passes through (2, –1)

   d. hyperbola with vertices (1, 2) and (5, 2) and the slope of one asymptote is $\frac{3}{2}$

   e. ellipse with vertices at (2, 1) and (6, 1); co-vertices at (4, 2) and (4, 0)

   f. hyperbola with vertices $(0,\pm2)$ and foci $(0,\pm4)$

   g. parabola with focus (5, 5), directrix: $y=-3$

   h. parabola with vertex (2, -1), passes through (4, 2), $p>0$, axis of symmetry: $x=2$

4. Solve the systems of equations by graphing.

a. $x^2 + y^2 = 16$
$x - y = 4$

b. $(x + 1)^2 + (y - 3)^2 = 1$
$x^2 + y^2 - 4x - 5 = 0$

c. $(x + 1)^2 + (y - 1)^2 = 1$
$(x - 2)^2 + (y - 1)^2 = 4$

5. Solve the systems algebraically.

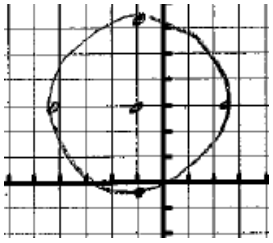
a. $x^2 + y^2 = 5$
$y = -x + 3$

b. $x^2 + y^2 = 9$
$x^2 + y^2 - 4x + 3 = 0$

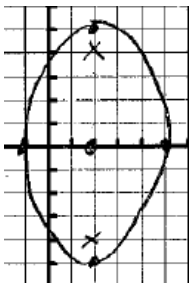
c. $4x^2 + 9y^2 - 36y = 0$
$x^2 + 9y - 27 = 0$

Answers

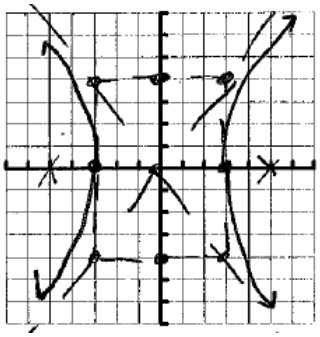
1a) c = (-1,3); r = $\sqrt{10} \approx 3.16$



1b) c = (2,0); v = (2,5), (2,-5); cv = (-1,0), (5,0); f = (2,4), (2,-4); ma = 10; mi = 6

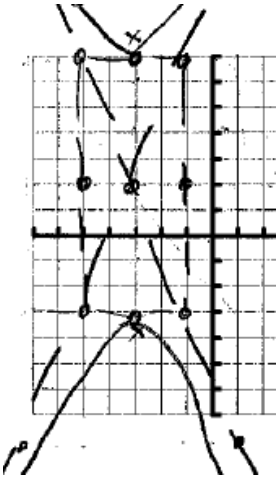


1c) c = (0,0); v = (-3,0), (3,0); f = (-5,0), (5,0); a = $y = \pm\frac{4}{3}x$



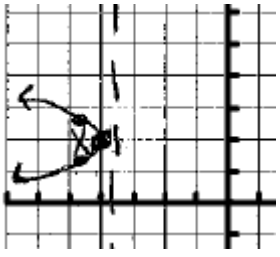
1d) c = (-3,2); v = (-3,-3), (-3,7); f = $\left(-3, 2 \pm \sqrt{29}\right)$

a: $y - 2 = \pm\frac{5}{2}(x + 3)$; t = 10

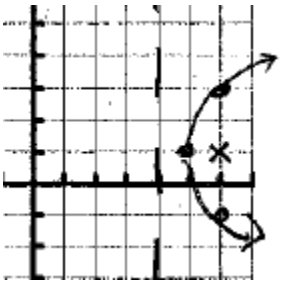


1e) v = (-4,2); f = $\left(-\frac{17}{4}, 2\right)$; d = $x = -\frac{15}{4}$;

lr = $\left(-\frac{17}{4}, \frac{3}{2}\right), \left(-\frac{17}{4}, \frac{5}{2}\right)$



1f) v = (5,1); f = (6,1); d = x = 4; lr = (6,3), (6,-1)



2a) Circle; $(x-3)^2+(y-1)^2=9$ 2b) Hyperbola; $\frac{x^2}{2}-\frac{y^2}{2}=1$ 2c) Ellipse; $\frac{(x+3)^2}{4}+\frac{(y-2)^2}{9}=1$

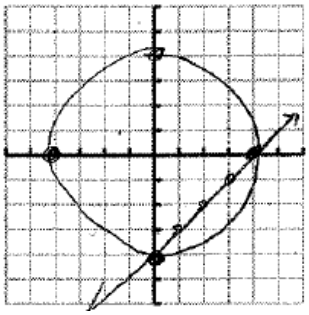
2d) Hyperbola; $\frac{(x+2)^2}{8}-\frac{(y+1)^2}{18}=1$ 2e) Parabola; $\left(x+\frac{1}{2}\right)^2=y+\frac{21}{4}$

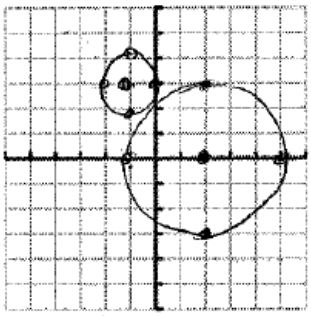
3a) $(x+2)^2+(y-3)^2=16$ 3b) $\frac{(x-3)^2}{16}+\frac{(y+4)^2}{4}=1$ 3c) $(x-1)^2+(y-4)^2=26$

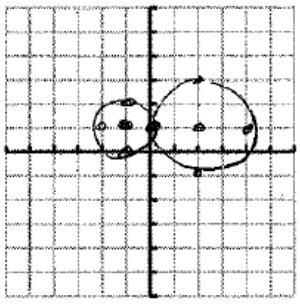
3d) $\frac{(x-3)^2}{4}-\frac{(y-2)^2}{9}=1$ 3e) $\frac{(x-4)^2}{4}+(y-1)^2=1$ 3f) $\frac{y^2}{4}-\frac{x^2}{12}=1$

3g) $(x-5)^2=16(y-1)$ 3h) $(x-2)^2=\frac{4}{3}(y+1)$

4a) (0,-4), (4,0) 4b) No solution 4c) (0,1)







5a) (2,1), (1,2) 5b) (3,0) 5c) (3,2), (-3,2)

PreCalculus
Review for Midterm Exam
Matrices

Given the following matrices, simplify the expressions, using fractions instead of decimals.

$A=\begin{bmatrix}-3 & 2\\ 0 & 5\end{bmatrix}$ $B=\begin{bmatrix}2 & -3\\ 4 & -1\end{bmatrix}$ $C=\begin{bmatrix}6 & -4\\ 3 & -2\end{bmatrix}$ $D=\begin{bmatrix}-1 & 4 & 0\\ 3 & -5 & 2\\ -4 & 3 & -2\end{bmatrix}$ $E=\begin{bmatrix}1 & 6 & -3\\ 2 & -4 & -1\end{bmatrix}$

1. 3(A – C)
2. |D|
3. – ½(ED)
4. C^{-1}
5. B^2
6. |B|
7. 2A – 3B + C
8. Evaluate by expansion by minors.

$$\begin{vmatrix}3 & 4 & -1\\ -2 & 3 & 0\\ 1 & 2 & 0\end{vmatrix}$$

9. Solve.

$$\begin{vmatrix}5 & 7x\\ -x & -6\end{vmatrix}=-2$$

10. Solve for x and y. $2\begin{bmatrix}x+2\\ y-3\end{bmatrix}+\begin{bmatrix}5\\ -4\end{bmatrix}=\begin{bmatrix}7\\ 1\end{bmatrix}$

11. Solve using a matrix equation.

$$2x+4y=-5$$
$$3x-7y=4$$

12. Solve for x and y.

$$\begin{bmatrix}x & -7\\ 3 & y\end{bmatrix}\begin{bmatrix}2\\ 5\end{bmatrix}=\begin{bmatrix}10\\ 1\end{bmatrix}$$

13. Multiply: $\begin{bmatrix}3 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}1 & 6\\ 2 & -1\end{bmatrix}$

14. Multiply: $\begin{bmatrix}1 & 5 & -4\\ 6 & 0 & -1\end{bmatrix}\begin{bmatrix}2 & -1\\ 3 & -3\\ 1 & 1\end{bmatrix}$

15. Find the inverse of

a) $\begin{bmatrix} 3 & -4 \\ 4 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 4 \\ -6 & -12 \end{bmatrix}$

16. You can only find the inverse of a _____ matrix.

17. If $A_{2x3} \cdot B_{3x1} = C$ find the dimensions of C. _____

Word Problems. (a) define the variables (b) write the system of equations
(c) write the matrix representation of the system (d) write your answer in a complete sentence.

18. The perimeter of a rectangular picture is 86 inches. Twice the width exceeds the length by 2 inches. What are the dimensions of the picture?

19. Mrs. Mardis buys 2 granola bars and 3 coffee's for $21.83. Mrs. Doyle buys 5 granola bars and 1 coffee for $15.90. How much does one granola bar and one coffee cost?

20. Your team goes to eat at a restaurant. There are 26 people eating dinner. Some team members order the buffet for $12.99 and some order the grilled steak meal for $15.95. Coach got the bill. It was $364.38. How many people ordered the buffet?

21. Ramona spent $17.00 on two different types of lollipops for Spring Fling prizes. Some cost $0.50 and some cost $0.35. If she bought a total of 40 lollipops, how many of each kind did she buy?

22. Flourish and Blotts store sells books. Some cost $6.00 and some cost $7.00. On Wednesday, Flourish and Blotts sold 27 books for $177.00. How many of each did they sell?

23. At a spring concert, tickets for adults cost $4.00 and tickets for students cost $2.50. How many of each kind of ticket were purchased if 125 tickets were bought for $413.

Answers

1. $\begin{bmatrix} -27 & 18 \\ -9 & 21 \end{bmatrix}$ 2. -12 3. $\begin{bmatrix} \frac{-29}{2} & \frac{35}{2} & -9 \\ 5 & \frac{-25}{2} & 3 \end{bmatrix}$ 4. Not possible

5. $\begin{bmatrix} -8 & -3 \\ 4 & -11 \end{bmatrix}$ 6. 10 7. $\begin{bmatrix} -6 & 9 \\ -9 & 11 \end{bmatrix}$ 8. 7

9. $x = \pm 2$ 10. $x = -1; y = \frac{11}{2}$ 11. $\left(\frac{-19}{26}, \frac{-23}{26}\right)$ 12. $x = \frac{45}{2}; y = -1$

13. $\begin{bmatrix} 1 & 19 \\ 4 & -2 \end{bmatrix}$ 14. $\begin{bmatrix} 13 & -20 \\ 11 & -7 \end{bmatrix}$ 15. a) $\begin{bmatrix} -\frac{1}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{3}{10} \end{bmatrix}$ b) Not possible 16. Square

17. 2 x 1

18. a) L = length, W = width b) $\begin{aligned} 2L+2W &= 86 \\ -L+2W &= 2 \end{aligned}$ c) $\begin{bmatrix} 2 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} L \\ W \end{bmatrix} = \begin{bmatrix} 86 \\ 2 \end{bmatrix}$

d) The length of the picture is 28 inches and the width is 15 inches.

19. a) g = granola, c = coffee b) $\begin{aligned} 2g+3c &= 21.83 \\ 5g+c &= 15.90 \end{aligned}$ c) $\begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} g \\ c \end{bmatrix} = \begin{bmatrix} 21.83 \\ 15.9 \end{bmatrix}$

d) The granola costs $1.99 and the coffee costs $5.95.

20. a) b= buffet, g = grilled steak b) $\begin{aligned} 12.99b+15.95g &= 364.38 \\ b+g &= 26 \end{aligned}$ c) $\begin{bmatrix} 12.99 & 15.95 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} b \\ g \end{bmatrix} = \begin{bmatrix} 364.38 \\ 26 \end{bmatrix}$

d) 17 people ordered the buffet.

21. a) x = lollipop 1, y = lollipop 2 b) $\begin{aligned} .5x+.35y &= 17 \\ x+y &= 40 \end{aligned}$ c) $\begin{bmatrix} .5 & .35 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ 40 \end{bmatrix}$

d) She bought 20 of each kind.

22. a) x = $6 dollar book, y = $7 book b) $\begin{aligned} 6x+7y &= 177 \\ x+y &= 27 \end{aligned}$ c) $\begin{bmatrix} 6 & 7 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 177 \\ 27 \end{bmatrix}$

d) They sold 12 of the $6 books and 15 of the $17 books

23. a) a = adults, s = students b) $\begin{aligned} 4a+2.5s &= 413 \\ a+s &= 125 \end{aligned}$ c) $\begin{bmatrix} 4 & 2.5 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a \\ s \end{bmatrix} = \begin{bmatrix} 413 \\ 125 \end{bmatrix}$

d) 67 adults and 58 student tickets were purchased.

PreCalculus
Review for Midterm Exam
Intro to Trig – Angles & SOHCAHTOA

1. In which quadrant does θ terminate if: (a) sine is negative and tangent is negative
(b) cosecant is positive and secant is negative
(c) cotangent is negative and cosine is positive

2. In which quadrant does each angle terminate? (a) $\frac{8\pi}{5}$ (b) -543° (c) $\frac{23\pi}{7}$ (d) 420°

3. Find a positive and negative angle that is coterminal with: (a) 73° (b) $\frac{4\pi}{7}$

4. Change each to degree measure: (a) $\frac{8\pi}{3}$ (b) $-\frac{4\pi}{15}$

5. Change to radian measure: (a) 125° (b) -540°

6. If $\sec x = \frac{8}{3}$ and x is an angle in Quadrant IV, find: (a) $\sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\csc x$ (e) $\cot x$

7. If (–5, –6) lies on the terminal side of angle A in standard position, find:

(a) $\sin A$ (b) $\cos A$ (c) $\tan A$ (d) $\csc A$ (e) $\sec A$ (f) $\cot A$

8. Find the reference angle for each: (a) 117° (b) $\frac{15\pi}{8}$ (c) -290° (d) $-\frac{29\pi}{11}$

9. For what angles between 0 and 2π is the secant function undefined?

10. Find the value of each function:

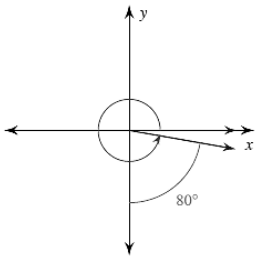
(a) $\cos 720^\circ$ (b) $\sec 150^\circ$ (c) $\sin(-60^\circ)$ (d) $\tan 225^\circ$ (e) $\csc 180^\circ$ (f) $\cot(-120^\circ)$

(g) $\cos\frac{8\pi}{3}$ (h) $\tan\left(-\frac{3\pi}{2}\right)$ (i) $\sec\frac{2\pi}{3}$ (j) $\sin\frac{13\pi}{6}$ (k) $\csc\frac{7\pi}{3}$ (l) $\cot 6\pi$

11. If $\csc\theta > 0$ and $\cot\theta < 0$, in which quadrant does θ terminate?

Find the measure of each angle.

12.



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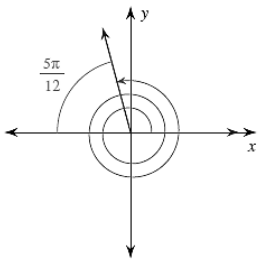

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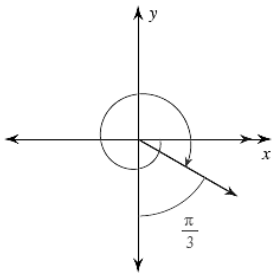


15. Find an angle between 0° and 360° that is coterminal with -435°

16. Find an angle between 0 and 2π that is coterminal with $\frac{11\pi}{3}$.

Find a positive and a negative coterminal angle for each given angle.

17. $\frac{-7\pi}{6}$

18. 640°

19. **Cos θ** $= \frac{2}{5}$, where **Sin θ > 0 and tan θ > 0.** Find the exact values of the five remaining trig functions.

Answers

1a) quadrant IV 1b) quadrant II 1c) quadrant IV

2a) quadrant IV 2B) quadrant II 2c) quadrant III 2d) quadrant I

3a) 433°, –287° 3b) $\frac{18\pi}{7}, \frac{-10\pi}{7}$

4a) 480° 4b) –48°

5a) $\frac{25\pi}{36}$ 5b) -3π

6a) $-\frac{\sqrt{55}}{8}$ 6b) $\frac{3}{8}$ 6c) $-\frac{\sqrt{55}}{3}$ 6d) $-\frac{8\sqrt{55}}{55}$ 6e) $-\frac{3\sqrt{55}}{55}$

7a) $-\frac{6\sqrt{61}}{61}$ 7b) $-\frac{5\sqrt{61}}{61}$ 7c) $\frac{6}{5}$ 7d) $-\frac{\sqrt{61}}{6}$ 7e) $-\frac{\sqrt{61}}{5}$ 7f) $\frac{5}{6}$

8a) $\theta' = 63°$ 8b) $\theta' = \frac{\pi}{8}$ 8c) $\theta' = 70°$ 8d) $\theta' = \frac{4\pi}{11}$ 9) $\frac{\pi}{2}, \frac{3\pi}{2}$

10a) 1 10b) $-\frac{2\sqrt{3}}{3}$ 10c) $-\frac{\sqrt{3}}{2}$ 10d) 1 10e) und. 10f) $\frac{\sqrt{3}}{3}$

10g) $-\frac{1}{2}$ 10h) und. 10i) -2 10j) $\frac{1}{2}$ 10k) $\frac{2\sqrt{3}}{3}$ 10l) und.

11) II 12) 350° 13) $\frac{55\pi}{12}$ 14) $\frac{-13\pi}{6}$ 15) 285° 16) $\frac{5\pi}{3}$

17) $\frac{5\pi}{6}$; $\frac{-19\pi}{6}$ 18) 280°; -80° 19) $\mathbf{Sin} = \frac{\sqrt{21}}{5}$; $\mathbf{Tan} = \frac{\sqrt{21}}{2}$

$\mathbf{Csc} = \frac{5\sqrt{21}}{21}$; $\mathbf{Sec} = \frac{5}{2}$; $\mathbf{Cot} = \frac{2\sqrt{21}}{21}$

Review for Midterm Exam
Triangle Trig – Right Triangles & Laws of Sines and Cosines

1. Solve triangle ABC if angle B is a right angle, side a = 11.1 and side b = 14.7.

2. Solve triangle ABC is angle A = 42.3°, side b = 6.1 and side c = 8.3.

3. Find the area of triangle ABC in question #2.

4. Solve triangle ABC if angle A = 82.1°, angle B = 38.7° and side a = 12.9.

5. Solve triangle ABC if side a = 9, side b = 11 and side c = 14.

6. Find the area of triangle ABC in question #5.

7. Find the angle of elevation to the top of a 10.3 foot tree that is casting a 28.9 foot shadow.

8. A ship leaves port and sails with a bearing of S 42° W at a speed of 23 mph. After 5 hours, how far south and how far west is the ship from the port?

9. At Pope High School, there is a flagpole mounted on the roof. From a point 200 feet in front of Pope, the angles of elevation to the base of the flagpole on the roof and to the top of the flagpole on the roof are 24° and 37° respectfully. Find the height of the flagpole.

10. From the top of a 55 foot tall lighthouse, a Coast Guard officer sights a boat in difficulty. The angle of depression to the boat is 6°. How far is the boat from the shoreline?

11. A 70 foot line is attached to a kite. When the kite has pulled the line tight, the angle of elevation of the kite is 56°. Find the height of the kite.

12. A plane is 250 miles north and 175 miles east of the airport. What bearing should the pilot follow if he wants to fly directly to the airport?

Solve the triangles using the Law of Sines, Law of Cosines or both.

13. a = 8.5 b = 12 A = 42°

14. C = 120° a = 4 b = 6

15. a = 55 b = 34 c = 70

16. C = 15° a = 6.25 b = 2.15

Use a calculator to evaluate each function. Round your answers to 4 decimal places.

17. sin 25°

18. cos 65°

19. cot 71.5°

20. sec 42°12’

21. cos 8°50’25”

22. tan $\pi/16$

23. csc 1.25

24. csc 0

Find the value of Θ in degrees. Round to the nearest hundredth.

25. sin Θ = 0.8191 26. cos Θ = 0.9848 27. tan Θ = 1.1920 28. sec Θ = 1.4123

Find the value of Θ in D°M'. Round to the nearest minute.

29. cos Θ = 0.4223 30. tan Θ = 1.5002 31. csc Θ = 1.5555 32. cot Θ = 2.1234

Answers

1) A = 49°, C = 41°, c = 9.6

2) B = 51.8° C = 85.9°, a = 5.6

3) 17.0 square units

4) C = 59.2°, b = 8.1, c = 11.2

5) option 1: A = 39.9°, B = 51.8°, C = 88.3°
option 2: A = 40°, B = 50.8°, C = 89.2°

6) 49.5 square units

7) 19.6°

8) south 85.5 miles, west 77 miles

9) 61.7 feet

10) 523.3 feet

11) 58 feet

12) W 55° S or S 35° W

13) B = 70.8°, C = 67.2°, c = 11.7
AND B = 109.2°, C = 28.8°, c = 6.1

14) options 1: c = 8.7, A = 23.5°, B = 36.5°
option 2: c = 8.7, A = 23.3°, B = 36.7°

15) C = 101.1°, A = 50.4, B = 28.5°

16) A = 157.4°, B = 7.6°, c = 4.2

17) 0.4226

18) 0.4226 19) 0.3346 20) 1.3499 21) 0.9881 22) 0.1989

23) 1.0538 24) undefined 25) 54.99° 26) 10° 27) 50.01°

28) 44.92° 29) 65°1' 30) 56°19' 31) 40°0' 32) 25°13'

Review for Midterm Exam
Graphs of Trig Functions & Sinusoidal Models

1. Given: The domain of a sinusoid is $[-82^0, 998^0]$

   (a) The phase shift of the function is _____ (b) The period of the function is _____

2. Given: The range of a sinusoidal function is $[-24, -7]$. Find the following:

   (a) amplitude of the function _____ (b) vertical shift of the function _____

Find the amplitude, period, vertical shift, and phase shift of the following.

3. $y = 3\sin\left(\frac{x}{4} + \frac{\pi}{12}\right) - 1$

4. $y = -2\cos(3\theta - 120°) + 2$

Graph each of the following.

5. $y = -2\sin\left(\frac{1}{3}x + \frac{\pi}{6}\right) + 1$

6. $y = 3\cos 2(\theta - 60°) - 2$

7. $y = 4\cos(2\theta)$

8. $y = \sin\left(3x - \frac{3\pi}{2}\right)$

9. $y = -2\cos\left(\frac{1}{2}x + \frac{5\pi}{6}\right) - 2$

10. $y = 3\sin(3\theta - 45°)$

Write the equation of each function.

11. Write the equation of a cosine function that has amplitude 5, period 270°, phase shift 60°, and vertical shift 3.

12. Write the equation of a sine graph with amplitude 2, period π, phase shift $-\frac{\pi}{4}$, and vertical shift -4.

13. Write the equation of a sine function whose domain is $[8^0, 68^0]$ and whose range is $[4, 7]$.

Answers

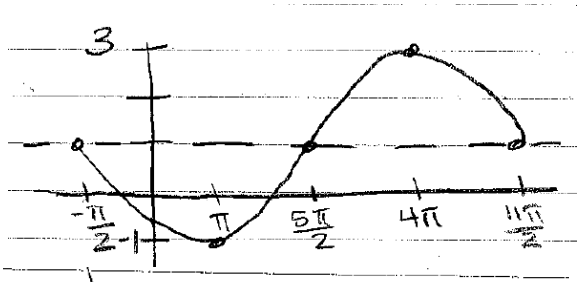
1a) $-82°$ 1b) $1080°$

2a) 8.5 2b) -15.5

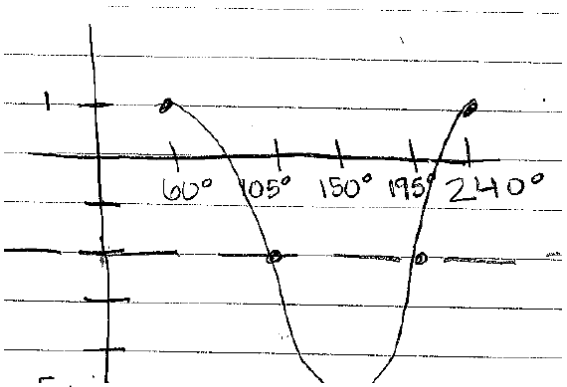
3) amp = 3 period = 8π vs = -1 ps = $-\frac{\pi}{3}$

4) amp = 2 period = $120°$ vs = 2 ps = $40°$

5)



6)



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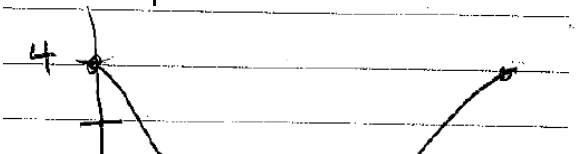

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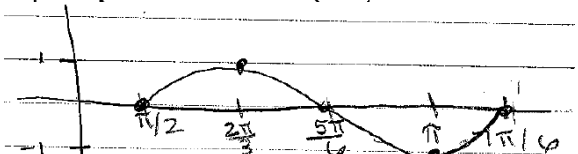

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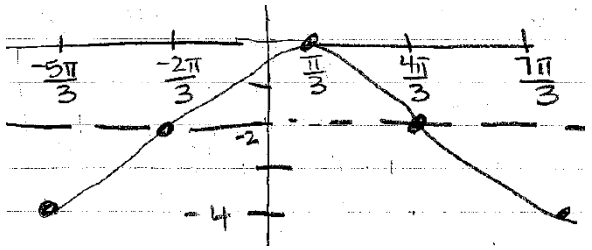

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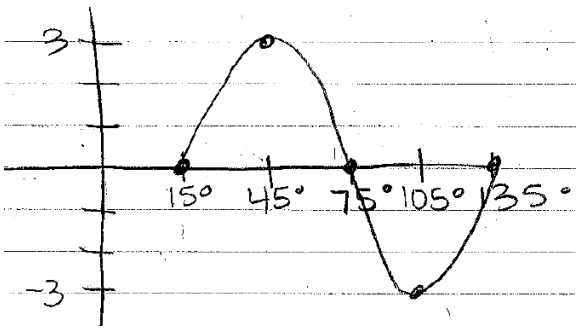

11) $y = 5\cos\frac{4}{3}(\theta - 60°) + 3$

12) $y = 2\sin\left(2\left(x + \frac{\pi}{4}\right)\right) - 4$

13) $y = 1.5\sin 6(\theta - 8°) + 5.5$