PreCalculus

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More Sum & Difference Identities WS

1. Using angles from the unit circle, find the EXACT value of $\cos 255^0$.

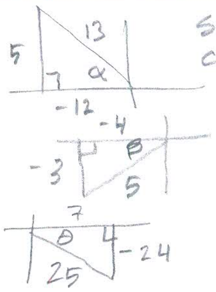
$$\cos(210^\circ + 45^\circ) = \cos 210^\circ \cos 45^\circ - \sin 210^\circ \sin 45^\circ$$
$$= \left(-\tfrac{\sqrt{3}}{2}\right)\left(\tfrac{\sqrt{2}}{2}\right) - \left(-\tfrac{1}{2}\right)\left(\tfrac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6}+\sqrt{2}}{4}$$

#2-6. Find the following given:

α is in quadrant II and $\csc\alpha = \dfrac{13}{5}$

β is in quadrant III and $\cot\beta = \dfrac{4}{3}$

θ is in quadrant IV and $\sec\theta = \dfrac{25}{7}$



$\sin\alpha = 5/13$
$\cos\alpha = -12/13$

$\sin\beta = -3/5$
$\cos\beta = -4/5$

$\sin\theta = -24/25$
$\cos\theta = 7/25$

2. $\cos(\beta-\alpha)$

$$= \cos\beta\cos\alpha + \sin\beta\sin\alpha$$
$$= \left(-\tfrac{4}{5}\right)\left(-\tfrac{12}{13}\right) + \left(-\tfrac{3}{5}\right)\left(\tfrac{5}{13}\right) = \frac{48-15}{65} = \frac{33}{65}$$

3. $\sin(\alpha+\beta)$

$$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\tfrac{5}{13}\right)\left(-\tfrac{4}{5}\right) + \left(-\tfrac{12}{13}\right)\left(-\tfrac{3}{5}\right) = \frac{-20+36}{65} = \frac{16}{65}$$

4. $\tan(\beta+\theta)$

$$= \frac{\tan\beta + \tan\theta}{1 - \tan\beta\tan\theta} = \frac{\frac{3}{4} + \frac{-24}{7}}{1-\left(\frac{3}{4}\right)\left(\frac{-24}{7}\right)} = \frac{\frac{-75}{28}}{\frac{100}{28}} = \frac{-75}{100} = -\frac{3}{4}$$

5. $\sin\left(\theta - \dfrac{7\pi}{6}\right)$

$$= \sin\theta\cos\tfrac{7\pi}{6} - \cos\theta\sin\tfrac{7\pi}{6}$$
$$= \left(-\tfrac{24}{25}\right)\left(-\tfrac{\sqrt{3}}{2}\right) - \left(\tfrac{7}{25}\right)\left(-\tfrac{1}{2}\right) = \frac{24\sqrt{3}+7}{50}$$

6. $\cos\left(\dfrac{5\pi}{3} + \alpha\right)$

$$= \cos\tfrac{5\pi}{3}\cos\alpha - \sin\tfrac{5\pi}{3}\sin\alpha$$
$$= \left(\tfrac{1}{2}\right)\left(-\tfrac{12}{13}\right) - \left(-\tfrac{\sqrt{3}}{2}\right)\left(\tfrac{5}{13}\right) = \frac{-12+5\sqrt{3}}{26}$$

7. Simplify: $\sin\left(\dfrac{3\pi}{2} + x\right)$

$$= \sin\tfrac{3\pi}{2}\cos x + \cos\tfrac{3\pi}{2}\sin x$$
$$= -1\cos x + 0\cdot\sin x = -\cos x$$

#8-10. Solve each of the following equations over the interval $[0, 2\pi)$

8. $\cos\left(\frac{5\pi}{4} - x\right) = \cos\left(\frac{5\pi}{4} + x\right) - 1$

$$\cancel{\cos\frac{5\pi}{4}\cos x} + \sin\frac{5\pi}{4}\cdot\sin x = \cancel{\cos\frac{5\pi}{4}\cdot\cos x} - \sin\frac{5\pi}{4}\cdot\sin x - 1$$

$$2\cdot\sin\frac{5\pi}{4}\cdot\sin x = -1$$

$$2\left(-\frac{\sqrt{2}}{2}\right)\cdot\sin x = -1$$

$$-\sqrt{2}\sin x = -1$$

$$\rightarrow \sin x = \frac{1}{\sqrt{2}}$$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}}$$

9. $2\cos\left(x - \frac{3\pi}{2}\right) = \cot\frac{\pi}{6}$

$$2\left(\cancel{\cos x \underset{0}{\cos\frac{3\pi}{2}}} + \sin x\cdot\underset{(-1)}{\sin\frac{3\pi}{2}}\right) = \sqrt{3}$$

$$-2\sin x = \sqrt{3}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{4\pi}{3}, \frac{5\pi}{3}}$$

10. $\sin\left(x + \frac{7\pi}{2}\right) + 5 - 5\cos x = 8\cos^2 x$

$$\cancel{\sin x \underset{0}{\cos\frac{7\pi}{2}}} + \cos x\,\underset{(-1)}{\sin\frac{7\pi}{2}} + 5 - 5\cos x = 8\cos^2 x$$

$$-\cos x + 5 - 5\cos x = 8\cos^2 x$$

$$0 = 8\cos^2 x + 6\cos x - 5$$

$$(4\cos x + 5)(2\cos x - 1)$$

$$\cancel{\cos x = \frac{-5}{4}} \qquad \cos x = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$