(Purple Book)

Multiple Angle Practice Exercises

21) $\sin^2 x = 3(1-\sin^2 x)$

$\sin^2 x = 3 - 3\sin^2 x$

$4\sin^2 x = 3$

$\sin^2 x = 3/4$

$\sin x = \pm \sqrt{3}/2 \longrightarrow x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

22) $\tan 3x (\tan x - 1) = 0$

$\tan 3x = 0$

① $3x = 0, \pi$

② $2\pi, 3\pi$

③ $4\pi, 5\pi$

$\Big\} x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\tan x - 1 = 0$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

23) $3\tan^2 x - 1 = 0$

$\tan x = \pm 1/\sqrt{3}$

$x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

24) $\cos 2x = 0$

① $2x = \pi/2, 3\pi/2$

② $5\pi/2, 7\pi/2$

$\Big\} x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

$\cos x = -1/2$

$x = 2\pi/3, 4\pi/3$

25) $\cos x (\cos^2 x - 1) = 0$

$\cos x = 0 \quad \cos x = \pm 1 \longrightarrow x = 0, \pi/2, \pi, 3\pi/2$

27) $\tan x (3\tan^2 x - 1) = 0$

$\tan x = 0 \quad \tan x = \pm \frac{1}{\sqrt{3}} \longrightarrow x = 0, \pi, \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

28) $2(1-\cos^2 x) = 2 + \cos x$

$2 - 2\cos^2 x = 2 + \cos x$

$2\cos^2 x + \cos x = 0$

$\cos x (2\cos x + 1) = 0$

$\cos x = 0 \quad \cos x = -\frac{1}{2} \longrightarrow x = \pi/2, 3\pi/2, 2\pi/3, 4\pi/3$

31) $2\sin x + \frac{1}{\sin x} = 0$

$2\sin^2 x + 1 = 0$

$\sin^2 x = -1/2 \longrightarrow$ no solution (imaginary)

32) $\sin 2x = -\frac{\sqrt{3}}{2}$

① $2x = 4\pi/3, 5\pi/3$

② $10\pi/3, 11\pi/3$

$\Big\} \; x = 2\pi/3, 5\pi/6, 5\pi/3, 11\pi/6$

33) $\left(\frac{1}{\sin x} + \frac{\cos x}{\sin x} = 1\right) \sin x$

$1 + \cos x = \sin x$

$1 + 2\cos x + \cos^2 x = \sin^2 x$

$1 + 2\cos x + \cos^2 x = 1 - \cos^2 x$

$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$\cos x = 0 \quad \cos x = -1 \longrightarrow x = \frac{\pi}{2}, \cancel{\pi}, \frac{3\pi}{2}$

34) $\tan 3x = 1$

① $3x = \pi/4, 5\pi/4$

② $9\pi/4, 13\pi/4$

③ $17\pi/4, 21\pi/4$

$\Big\} \; x = \pi/12, 5\pi/12, 3\pi/4, 13\pi/12, 17\pi/12, 7\pi/4$

36) $\sec 4x = 2$

$\cos 4x = 1/2$

① $4x = \pi/3, 5\pi/3$

② $7\pi/3, 11\pi/3$

③ $13\pi/3, 17\pi/3$

④ $19\pi/3, 23\pi/3$

$\Big\} \; x = \pi/12, 5\pi/12, 7\pi/12, 11\pi/12, 13\pi/12, 17\pi/12, 19\pi/12, 23\pi/12$

37) $\left(\frac{1+\cos x}{1-\cos x} = 0\right)(1-\cos x)$

$1+\cos x = 0$

$\cos x = -1 \longrightarrow x = \pi$

38) $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2} \quad \sin x = -1 \longrightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

39) $2\sec^2 x + \tan^2 x - 3 = 0$

$2(\tan^2 x + 1) + \tan^2 x - 3 = 0$

$2\tan^2 x + 2 + \tan^2 x - 3 = 0$

$3\tan^2 x - 1 = 0$

$\tan^2 x = 1/3$

$\tan x = \pm 1/\sqrt{3} \longrightarrow x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

40) $\cos x + \sin x \cdot \tan x = 2$

$\left(\cos x + \frac{\sin^2 x}{\cos x} = 2\right)\cos x$

$\cos^2 x + \sin^2 x = 2\cos x$

$1 = 2\cos x$

$\cos x = 1/2 \longrightarrow \pi/3, 5\pi/3$