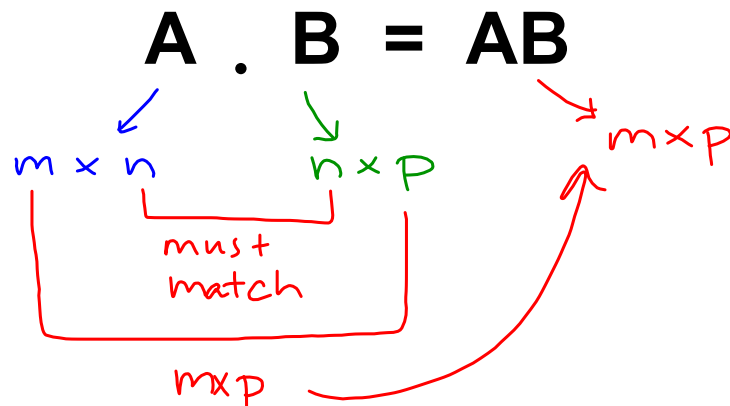


Multiplying Matrices



**1. State whether AB is defined.
If so, give the dimensions.**

a. $A: 2 \times 4$ $B: 4 \times 3$

$$A_{2 \times 4} \cdot B_{4 \times 3}$$

yes $A \cdot B$ is 2×3

b. $A: 1 \times 4$ $B: 1 \times 4$

$$A_{1 \times 4} \cdot B_{1 \times 4}$$

no $A \cdot B$ does not exist

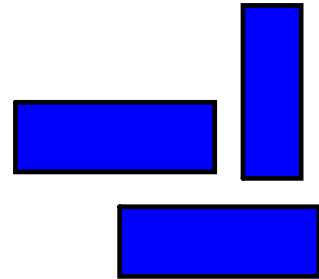
2. Find AB.

$$A = \begin{bmatrix} -1 & 5 \\ 5 & 2 \\ 0 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & -3 \\ 6 & 8 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 5 \\ 5 & 2 \\ 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -3 \\ 6 & 8 \end{bmatrix}$$

3×2 2×2

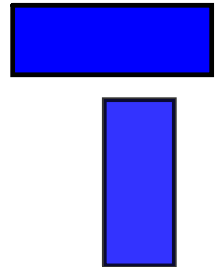


$$= \begin{bmatrix} \frac{-4+30}{20+12} & \frac{3+40}{-15+16} \\ \frac{0+-24}{0+-32} & \end{bmatrix} = \begin{bmatrix} 26 & 43 \\ 32 & 1 \\ -24 & -32 \end{bmatrix}$$

3. Find AB and BA.

$$A = \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix}$$



$$AB = \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{-16+1}{0+-2} & \frac{-12+2}{0+-4} \\ \end{bmatrix} = \begin{bmatrix} -15 & -10 \\ -2 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} \frac{-16+0}{4+0} & \frac{-4+6}{1+-4} \\ \end{bmatrix} = \begin{bmatrix} -16 & 2 \\ 4 & -3 \end{bmatrix}$$

★ Matrix multiplication is Not commutative!

Properties of Matrix Operations

Associative Property of Addition $(A + B) + C = A + (B + C)$

Commutative Property of Addition $A + B = B + A$

Distributive Property with a scalar $c(A + B) = cA + cB$

Properties of Matrix Multiplication

Associative Property of Matrix Multiplication $A(BC) = (AB)C$

Left Distributive $A(B + C) = AB + AC$

Right Distributive $(A + B)C = AC + BC$

Associative Property of scalar multiplication $c(AB) = (cA)B = A(cB)$

4. Find $B(A + C)$ and $BA + BC$.

$$A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$B(A + C) = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \left(\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -12 & -9 \end{bmatrix}$$

$$BA + BC = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 4 \\ -8 & -2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -4 & -7 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -12 & -9 \end{bmatrix}$$

Same!