Multiplying Matrices

$$A \cdot B = AB$$

$m \times n$ $\quad$ $n \times p$ $\quad$ $m \times p$

must match

$m \times p$

1. **State whether AB is defined.**
   If so, give the dimensions.

a. A: 2 x 4 B: 4 x 3

$A_{2\times4} \cdot B_{4\times3}$

yes

$A \cdot B$ is 2×3

b. A: 1 x 4 B: 1 x 4

$A_{1\times4} \cdot B_{1\times4}$

no

$A \cdot B$ does not exist

2. Find AB.

$$A=\begin{bmatrix} -1 & 5 \\ 5 & 2 \\ 0 & -4 \end{bmatrix} \qquad B=\begin{bmatrix} 4 & -3 \\ 6 & 8 \end{bmatrix}$$

$$AB=\begin{bmatrix} -1 & 5 \\ 5 & 2 \\ 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -3 \\ 6 & 8 \end{bmatrix}$$

$3 \times 2 \qquad 2 \times 2$

$$=\begin{bmatrix} -4+30 & 3+40 \\ 20+12 & -15+16 \\ 0+-24 & 0+{}^{-}32 \end{bmatrix} = \begin{bmatrix} 26 & 43 \\ 32 & 1 \\ -24 & -32 \end{bmatrix}$$

3. Find AB and BA.

$$A=\begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix} \qquad B=\begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$AB=\begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -16+1 & -12+2 \\ 0+-2 & 0+-4 \end{bmatrix} = \begin{bmatrix} -15 & -10 \\ -2 & -4 \end{bmatrix}$$

$$BA=\begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -16+0 & -4+6 \\ 4+0 & 1+-4 \end{bmatrix} = \begin{bmatrix} -16 & 2 \\ 4 & -3 \end{bmatrix}$$

☆ Matrix multiplication is NOT commutative!

Properties of Matrix Operations

Associative Property of Addition $(A + B) + C = A + (B + C)$

Commutative Property of Addition $A + B = B + A$

Distributive Property with a scalar $c(A + B) = cA + cB$

Properties of Matrix Multiplication

Associative Property of Matrix Multiplication $A(BC) = (AB)C$

Left Distributive $A(B + C) = AB + AC$

Right Distributive $(A + B)C = AC + BC$

Associative Property of scalar multiplication $c(AB) = (cA)B = A(cB)$

4. Find B(A + C) and BA + BC.

$$A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$B(A+C) = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}\left(\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -12 & -9 \end{bmatrix}$$

$$BA + BC = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ -8 & -2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -4 & -7 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -12 & -9 \end{bmatrix}$$

Same!