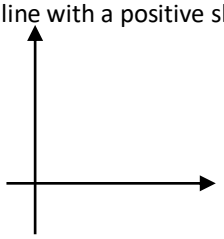
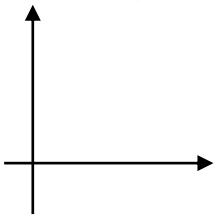
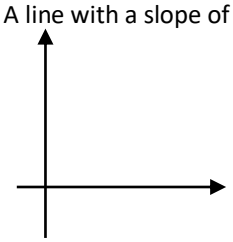
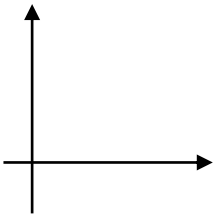
Parallel and Perpendicular Lines

The ***slope*** of a line is a number that measures how steep the line is.

- A ***horizontal*** line (a line of the form $y = c$) has a slope of zero.
- A ***vertical*** line (a line of the form $x = c$) has an undefined slope.
- Lines that are not horizontal or vertical may have a positive slope or a negative slope.

Draw the following:

A line with a positive slope.	A line with a negative slope.	A line with a slope of 0.	A line with an undefined slope.
			

The slope of a line is represented by the letter ***m***. No one knows for certain why ***m*** was chosen, but one theory is that it comes from "*monter*", which means "to climb" in French.

You have already learned the slope formula:

Given two points on a line, (x_1, y_1) and (x_2, y_2),
the slope of the line is calculated by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines and their slopes are related.

- Two lines are ***parallel*** if they are in the same plane but never intersect.
- Two non-vertical lines are ***parallel*** if they have the same slope.
- Two lines are ***perpendicular*** if they intersect at a 90° angle.
- Two non-vertical lines are ***perpendicular*** if the product of their slopes is –1.
  (In other words, two lines are ***perpendicular*** if the slopes are ***opposite reciprocals*** of each other.)

Find the slope of each line using the slope formula. Show clear work!
Determine whether each pair of lines is parallel, perpendicular, or neither.

A(1, 2) B(3, –4) C(9, –2) D(10, –5) E(9, –8) F(– 3, –12)

1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$

2. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$

3. $\overleftrightarrow{BC}$ and $\overleftrightarrow{EF}$

Algebraic Forms of a Line:

Slope-intercept form of a line is $y = mx + b$, where ***m*** is the slope and ***b*** is the y-intercept of the line.	Point-slope form of a line is $y - y_1 = m(x - x_1)$, where ***m*** is the slope and (x_1, y_1) is a point on the line.

Lines Parallel to Given Line

The line ***parallel*** to a given line through a given point will have the ***same slope*** as the given line, but a ***different y-intercept***. Use the slope of the given line, the given point, and the point-slope formula to write the equation.

Lines Perpendicular to Given Line

The line ***perpendicular*** to a given line through a given point will have a slope that is the ***opposite reciprocal*** of the given line. Use this information, the given point and the point-slope formula to write the equation.

Examples

4. Write an equation of the line that passes through (–3, 5) and is parallel to y = 2x – 4.

step 1 → substitute $m = 2$, and $(x_1, x_2) = (-3, 5)$ into the point-slope formula:

$y -$ _____ = _____ ($x -$ _____)

step 2 → distribute and put into slope-intercept form:

$y =$ _____

5. Consider the line 4x – 2y = –5.

Part a: What is the slope of the line? (Hint: Put the line into slope-intercept form.)

Part b: What is the slope of a line ***parallel*** to the line in part a?

Part c: What is the slope of a line ***perpendicular*** to the line in part a?

6. Consider the line 2x + 3y = 12.

Part a: What is the slope of the line?

Part b: What is the slope of a line ***perpendicular*** to the line in part a?

Part b: Write an equation of the line perpendicular to 2x + 3y = 12 that passes through (–4, 6).

Answers:
1. neither 2. perpendicular 3. parallel 4. y = 2x + 11 5. 2; 2; –½ 6. –2/3; 3/2; y = 3/2x + 12