Standards

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

The Complex Number System (N.CN) ***Use properties of rational and irrational numbers.***

MGSE9-12.N.CN.3 Find the conjugate of a complex number; use the conjugate to find the absolute value (modulus) and quotient of complex numbers.

Represent complex numbers and their operations on the complex plane.

MGSE9-12.N.CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

MGSE9-12.N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.*

MGSE9-12.N.CN.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Vector and Matrix Quantities (N.VM)

Represent and model with vector quantities.

MGSE9-12.N.VM.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $|\mathbf{v}|$, $||\mathbf{v}||$, v).

MGSE9-12.N.VM.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

MGSE9-12.N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

MGSE9-12.N.VM.4 Add and subtract vectors.

Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

Understand vector subtraction ***v*** – ***w*** as ***v*** + (– ***w***), where (– ***w***) is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

MGSE9-12.N.VM.5 Multiply a vector by a scalar.

Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise,

e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

Compute the magnitude of a scalar multiple c ***v*** using ||c ***v*** || = |c| ***v***. Compute the direction of c ***v*** knowing that when |c| ***v*** ≠ 0, the direction of c ***v*** is either along ***v*** (for c > 0) or against ***v*** (for c < 0).

Perform operations on matrices and use matrices in applications.

MGSE9-12.N.VM.6 Use matrices to represent and manipulate data, e.g., transformations of vectors.

MGSE9-12.N.VM.7 Multiply matrices by scalars to produce new matrices.

MGSE9-12.N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

MGSE9-12.N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

Perform operations on matrices and use matrices in applications continued

MGSE9-12.N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

MGSE9-12.N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

MGSE9-12.N.VM.12 Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Reasoning With Equations and Inequalities **(A.REI)*****Solve systems of equations***

MGSE9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

MGSE9-12.A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.

MGSE9-12.A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Interpreting Functions (F.IF)

Interpret functions that arise in applications in terms of the context

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Analyze functions using different representations

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

e. Graph trigonometric functions, showing period, midline, and amplitude.

Building Functions (F.BF)

Build new functions from existing functions

MGSE9-12.F.BF.4 Find inverse functions.

d. Produce an invertible function from a non-invertible function by restricting the domain.

Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle

MGSE9-12.F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

MGSE9-12.F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

MGSE9-12.F.TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number.

MGSE9-12.F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

MGSE9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

MGSE9-12.F.TF.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

MGSE9-12.F.TF.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

GSE9-12.F.TF.8 Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to find sin A, cos A, or tan A, given sin A, cos A, or tan A, and the quadrant of the angle.

MGSE9-12.F.TF.9 Prove addition, subtraction, double, and half-angle formulas for sine, cosine, and tangent and use them to solve problems.

Similarity, Right Triangles, and Trigonometry (G.SRT)

Apply trigonometry to general triangles

MGSE9-12.G.SRT.9 Derive the formula A = (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

MGSE9-12.G.SRT.10 Prove the Laws of Sines and Cosines and use them to solve problems.

MGSE9-12.G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations (G.GPE)

Translate between the geometric description and the equation for a conic section

MGSE9-12.G.GPE.2 Derive the equation of a parabola given a focus and directrix.

MGSE9-12.G.GPE.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Conditional Probability and the Rules of Probability (S.CP)

Use the rules of probability to compute probabilities of compound events in a uniform probability model

MGSE9-12.S.CP.8 Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]x[P(B|A)] = [P(B)]x[P(A|B)], and interpret the answer in terms of the model.

MGSE9-12.S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

Use Probability to Make Decisions (S.CP)

Calculate expected values and use them to solve problems

MGSE9-12.S.MD.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

MGSE9-12.S.MD.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

MGSE9-12.S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

MGSE9-12.S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decision

MGSE9-12.S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

- Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
- Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

MGSE9-12.S.MD.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

MGSE9-12.S.MD.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

The following enrichment standards to the Pre-Calculus and Accelerated Pre-Calculus Georgia Standards of Excellence were based upon:

Student preparation for AP Calculus courses in preparation for the study of advanced algebraic structures . Such advanced topics include investigation of sequential structures and sums.
Collaboration of the Cobb County School District Pre-Calculus teachers.

CSE9-12.N.SEQ.1. Students will recognize, formulate, and use sequences and series.

Formulate and write recursive rules for numeric patterns.
Formulate and write explicit rules for numeric patterns.
Recognize and write formulas for arithmetic and geometric sequences.
Identify arithmetic and geometric sequences as discrete linear and exponential patterns.
Explore the partial sums of arithmetic and geometric sequences.
Define series as the sum of a sequence and be able to write as a formula using summation (sigma) notation.
Explore the limits of infinite geometric series.
Explore and use formulas for sums of polynomial sequences.
Prove summation formulas using mathematical induction.

CSE9-12.A.REI.1. Students will solve trigonometric equations both graphically and algebraically.

CSE9-12.N.CN.1. Students will use complex numbers in polar (trigonometric) form.

Represent complex numbers in both rectangular and polar (trigonometric) forms.
Evaluate the products, quotients, powers, and roots of complex numbers in polar (trigonometric) form.

CSE9-12.F.POL.1. Students will understand and explore the polar coordinate plane.

Plot coordinates in the polar plane.
Express coordinates in both polar and rectangular forms.
Identify and use symmetry to graph standard polar equations.
Identify, and graph conic sections in polar form.
Express equations in both polar and rectangular forms.

CSE9-12.F.POL.2. Students will explore and use parametric equations.

Understand the use of a parameter.
Express equations in both parametric and rectangular forms.
Graph parametric equations as plane curves.
Model real-world situations using parametric equations and be able to determine quantities using parametric models.
Understand and apply limits to the parameter for a given parametric set.

CSE – Cobb Standard of Excellence

SEQ – Sequences and Series sub strand

POL – Polar and Parametric sub strand