The Rational Root Theorem and
The Fundamental Theorem of Algebra

The Rational Root Theorem

If $f(x) = a_n x^n + \ldots + a_1 x + a_0$ has integer coefficients, then every rational zero of $f(x)$ has the form:

$$\frac{p}{q} = \pm\frac{\text{factors of } a_0}{\text{factors of } a_n}$$

Example 1:

List the possible rational zeros of $f(x) = x^3 - 4x^2 - 11x + 30$. Find the zeros.

$\frac{p}{q}$: $\pm(1, 2, 3, 5, 6, 10, 15, 30)$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -11 & 30 \\ & & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5$ $\quad x = -3$

$\boxed{\{-3, 2, 5\}}$

Example 2:

List the possible rational zeros of $f(x) = 15x^4 - 68x^3 - 7x^2 + 24x - 4$. Find the zeros.

$$\frac{p}{q}: \pm\frac{1,\ 2,\ 4}{1,\ 3,\ 5,\ 15} = \pm\left(1, 2, 4, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{1}{15}, \frac{2}{15}, \frac{4}{15}\right)$$

$\frac{1}{5}$	15	−68	−7	24	−4	← degree 4
		3	−13	−4	4	
$-\frac{2}{3}$	15	−65	−20	20	0	← degree 3
		−10	50	−20		
	15	−75	30	0		← degree 2

$$15x^2 - 75x + 30 = 0$$

$$x^2 - 5x + 2 = 0$$

$$x = \frac{5 \pm \sqrt{17}}{2}$$

$$\left\{\frac{1}{5}, -\frac{2}{3}, \frac{5 \pm \sqrt{17}}{2}\right\}$$