Review - Sum & Difference Identities

1. $\sin(97^\circ + 43^\circ) = \sin 140^\circ$

2. $\cos(130^\circ - 72^\circ) = \cos 58^\circ$

3. $\tan(140^\circ - 60^\circ) = \tan 80^\circ$

4. $\sin\left(\frac{\pi}{5} - \frac{2\pi}{3}\right) = \sin\left(-\frac{7\pi}{15}\right) = -\sin\frac{7\pi}{15}$

5. $\cos\left(\frac{\pi}{6} + \frac{\pi}{7}\right) = \cos\frac{13\pi}{42}$

6. $\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \tan\frac{7\pi}{12}$

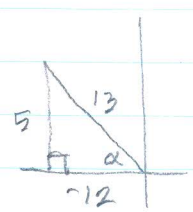
7. $\tan(-105^\circ) = \tan(45^\circ - 150^\circ) = \dfrac{\tan 45^\circ - \tan 150^\circ}{1 + \tan 45^\circ \tan 150^\circ}$

$= \dfrac{1 - -\frac{\sqrt{3}}{3}}{1 + -\frac{\sqrt{3}}{3}} = \dfrac{(3+\sqrt{3})}{(3-\sqrt{3})}\dfrac{(3+\sqrt{3})}{(3+\sqrt{3})} = \dfrac{12+6\sqrt{3}}{6} = \boxed{2+\sqrt{3}}$

8. $\sin 345^\circ = \sin(300^\circ + 45^\circ) = \sin 300^\circ \cos 45^\circ + \cos 300^\circ \sin 45^\circ$

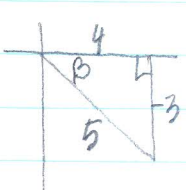
$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \dfrac{-\sqrt{6}+\sqrt{2}}{4} = \boxed{\dfrac{\sqrt{2}-\sqrt{6}}{4}}$



9. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

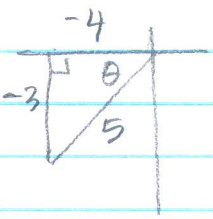
$= \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) - \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right) = \dfrac{20-36}{65} = \boxed{-\dfrac{16}{65}}$



10. $\cos(\beta + \alpha) = \cos\beta\cos\alpha - \sin\beta\sin\alpha$

$= \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right) = \dfrac{-48+15}{65} = \boxed{-\dfrac{33}{65}}$

11. $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \dfrac{-\frac{5}{12} - \left(-\frac{3}{4}\right)}{1 + \left(-\frac{5}{12}\right)\left(-\frac{3}{4}\right)} = \dfrac{\frac{16}{48}}{\frac{63}{48}} = \boxed{\dfrac{16}{63}}$



12. $\cos\left(\theta+\frac{\pi}{3}\right) = \cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}$
$= \left(-\frac{4}{5}\right)\left(\frac{1}{2}\right) - \left(-\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{-4+3\sqrt{3}}{10}}$

13. $\tan(\theta+\theta) = \frac{\tan\theta+\tan\theta}{1-\tan^2\theta} = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{3}{4}\right)}{1-\frac{9}{16}} = \frac{\frac{6}{4}}{\frac{7}{16}} = \boxed{\frac{24}{7}}$

14. $\sin(\pi-x) = \sin\pi\cos x - \cos\pi\sin x$
$= 0\cdot\cos x - (-1)\sin x = \sin x$ ✓

15. $\sin\left(\frac{3\pi}{2}+x\right) = \sin\frac{3\pi}{2}\cos x + \cos\frac{3\pi}{2}\sin x$
$= (-1)\cos x + 0\cdot\sin x = -\cos x$ ✓

16. $\cos(30°-x) + \cos(30°+x)$
$= \cos 30°\cos x + \sin 30°\sin x + \cos 30°\cos x - \sin 30°\sin x$
$= 2\cos 30°\cos x = 2\left(\frac{\sqrt{3}}{2}\right)\cos x = \sqrt{3}\cos x$ ✓

17. $\frac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta} = \frac{\sin\beta\cos\alpha - \cos\beta\sin\alpha}{\sin\alpha\sin\beta} = \frac{\cos\alpha}{\sin\alpha} - \frac{\cos\beta}{\sin\beta}$
$= \cot\alpha - \cot\beta$ ✓

18. $\cos(\alpha+\beta) + \cos(\alpha-\beta)$
$= \cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$= 2\cos\alpha\cos\beta$ ✓

19. $\sin x\cos\frac{\pi}{6} + \cos x\sin\frac{\pi}{6} - \left(\sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6}\right) = \frac{1}{2}$
$2\cos x\sin\frac{\pi}{6} = \frac{1}{2}$
$2\cos x\left(\frac{1}{2}\right) = \frac{1}{2}$ → $\cos x = \frac{1}{2}$
$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

20. $\frac{\tan x + \tan\pi}{1-\tan x\tan\pi} + 2(\sin x\cos\pi + \cos x\sin\pi) = 0$
$\tan x - 2\sin x = 0$
$\cos x\left(\frac{\sin x}{\cos x} - 2\sin x = 0\right)$ →
$\sin x - 2\sin x\cos x = 0$
$\sin x(1-2\cos x) = 0$
$\sin x = 0$ $\quad \cos x = \frac{1}{2}$
$\boxed{x = 0, \pi \quad x = \frac{\pi}{3}, \frac{5\pi}{3}}$

21. $\sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2} - (\cos x\cos\frac{\pi}{2} - \sin x\sin\frac{\pi}{2}) = 0$
$\cos x - \sin x = 0$
$\cos x = \sin x$
$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$