

Review w/ Double & Half Angles

$$1) \tan 45^\circ = \tan 2(22.5^\circ) = \frac{2 \tan 22.5^\circ}{1 - \tan^2 22.5^\circ} = \frac{2(1)}{1 - (1)^2} = \frac{2}{0} = \boxed{\text{undefined}}$$

$$2) \cos 2\left(\frac{4\pi}{3}\right) = 2\cos^2\left(\frac{4\pi}{3}\right) - 1 = 2\left(-\frac{1}{2}\right)^2 - 1 = \frac{1}{2} - \frac{2}{2} = \boxed{-\frac{1}{2}}$$

$$3) \sin 2(300^\circ) = 2\sin 300^\circ \cos 300^\circ = 2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{3}}{2} \quad \csc 600^\circ = \boxed{-\frac{2\sqrt{3}}{3}}$$

$$4) \sin\left(\frac{330^\circ}{2}\right) = +\sqrt{\frac{1 - \cos 330^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}} = \boxed{\frac{\sqrt{2 - \sqrt{3}}}{2}}$$

$$5) \cos\left(\frac{7\pi}{4}\right) = -\sqrt{\frac{1 + \cos \frac{7\pi}{4}}{2}} = -\sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2 + \sqrt{2}}{4}} = \boxed{-\frac{\sqrt{2 + \sqrt{2}}}{2}}$$

$$6) \cos\left(\frac{5\pi}{6}\right) = \sqrt{\frac{1 + \cos \frac{5\pi}{6}}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}}$$

$$\sec \frac{5\pi}{12} = \frac{4 \cdot (2 + \sqrt{3})}{\sqrt{(2 - \sqrt{3})(2 + \sqrt{3})}} = \frac{8 + 4\sqrt{3}}{\sqrt{4 - 3}} = \frac{\sqrt{8 + 4\sqrt{3}}}{\sqrt{1}} = \boxed{\sqrt{8 + 4\sqrt{3}}}$$

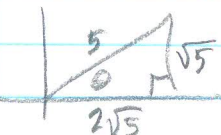
OR $\boxed{2\sqrt{2 + \sqrt{3}}}$

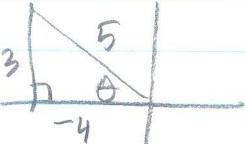
$$7) \begin{array}{|c|} \hline 24 \\ \hline \theta \\ \hline 25 \\ \hline \end{array} \quad \cos \theta = \frac{4}{25} \quad \cos 2\theta = \frac{1 + \cos \theta}{2} = \frac{1 + \frac{4}{25}}{2} = \frac{\frac{29}{25}}{2} = \frac{29}{50}$$

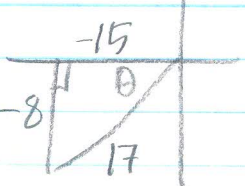
$$= \boxed{\frac{-7\sqrt{2}}{10}}$$

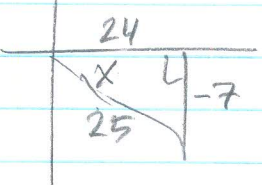
$$8) \begin{array}{|c|} \hline 3 \\ \hline \theta \\ \hline 1 \\ \hline \end{array} \quad \sin 2\theta = 2\sin \theta \cos \theta = 2\left(\frac{2\sqrt{2}}{3}\right)\left(\frac{1}{3}\right) = \boxed{\frac{4\sqrt{2}}{9}}$$

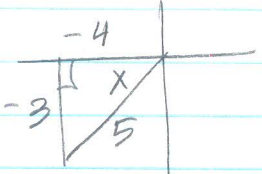
$$9) \begin{array}{|c|} \hline 4 \\ \hline \theta \\ \hline 5 \\ \hline \end{array} \quad \sin 2\theta = 2\sin \theta \cos \theta = 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{-\frac{24}{25}}$$

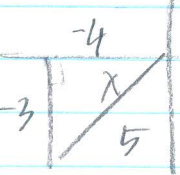
10)  $\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \frac{2\sqrt{5}}{5}}{2}} = \sqrt{\frac{5 - 2\sqrt{5}}{5} \cdot \frac{10}{20}} = \boxed{\frac{\sqrt{50 - 20\sqrt{5}}}{10}}$

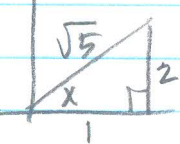
11)  $\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9}{5} \cdot \frac{1}{2}} = \boxed{\frac{3\sqrt{10}}{10}}$

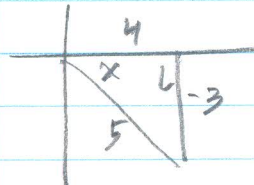
12)  $\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{1 + \frac{15}{17}}{-\frac{8}{17}} = \frac{32}{17} \left(\frac{17}{-8} \right) = \boxed{-4}$

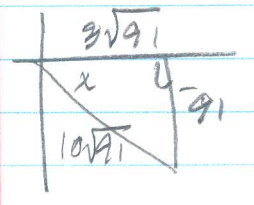
13)  $\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{1 - \frac{24}{25}}{-\frac{7}{25}} = \frac{1}{25} \left(\frac{25}{-7} \right) = \frac{-1}{7}$
 $\cot \frac{x}{2} = \boxed{-7}$

14)  $\sin 2x = 2 \sin x \cos x = 2 \left(\frac{-3}{5} \right) \left(\frac{-4}{5} \right) = \boxed{\frac{24}{25}}$

15)  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{2 \left(\frac{3}{4} \right)}{1 - \frac{9}{16}} = \frac{3}{2} \cdot \frac{16}{7} = \frac{24}{7}$
 $\cot 2x = \boxed{\frac{7}{24}}$

16)  $\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 - \frac{1}{\sqrt{5}}}{2}} = \sqrt{\frac{\sqrt{5} - 1}{\sqrt{5}} \cdot \frac{10}{20}} = \sqrt{\frac{5 - \sqrt{5}}{10} \cdot \frac{10}{10}} = \boxed{\frac{\sqrt{50 - 10\sqrt{5}}}{10}}$

17)  $\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{1 - \frac{4}{5}}{\frac{3}{5}} = \frac{1}{5} \cdot \left(\frac{5}{3}\right) = \boxed{\frac{-1}{3}}$

18)  $\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 - \frac{3}{10}}{2}} = \sqrt{\frac{\frac{7}{10}}{2}} = \sqrt{\frac{35}{100}} = \boxed{\frac{\sqrt{35}}{10}}$

19) $\cos 2x + \sin x = -2$
 $1 - 2\sin^2 x + \sin x = -2$
 $2\sin^2 x - \sin x - 3 = 0$
 $(2\sin x - 3)(\sin x + 1) = 0$
 ~~$\sin x = \frac{3}{2}$~~ $\sin x = -1$
 $x = \boxed{\frac{3\pi}{2}}$

20) $\cos 2x - \sin 2x = -2\sin x \cos x$
 $\cos 2x - 2\sin x \cos x = -2\sin x \cos x$
 $2\cos^2 x - 1 = 0$
 $\cos^2 x = \frac{1}{2}$
 $\cos x = \pm \frac{\sqrt{2}}{2}$
 $x = \boxed{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

21) $\cos^2 x - \frac{3}{2}(2\cos^2 x - 1) = 0$
 $-2\cos^2 x + \frac{3}{2} = 0$
 $\cos^2 x = \frac{3}{4}$
 $\cos x = \pm \frac{\sqrt{3}}{2}$
 $x = \boxed{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

22) $2\sin \frac{x}{2} = \sin x$
 $2 \sqrt{\frac{1 - \cos x}{2}} = \sin x$
 $2(1 - \cos x) = \sin^2 x$
 $2 - 2\cos x = 1 - \cos^2 x$
 $\cos^2 x - 2\cos x + 1 = 0$
 $(\cos x - 1)(\cos x - 1) = 0$
 $\cos x = 1$ $x = \boxed{0}$

23) $\sin^2 \frac{x}{2} = \cos^2 \frac{x}{2}$
 $\frac{1 - \cos x}{2} = \frac{1 + \cos x}{2}$
 $0 = 2\cos x$
 $0 = \cos x$
 $x = \boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

24) $\cos 2x - 11\cos x = 5$
 $2\cos^2 x - 1 - 11\cos x = 5$
 $2\cos^2 x - 11\cos x - 6 = 0$
 $(2\cos x + 1)(\cos x - 6) = 0$
 $\cos x = -\frac{1}{2}$ ~~$\cos x = 6$~~
 $x = \boxed{\frac{2\pi}{3}, \frac{4\pi}{3}}$

$$25) \cos\left(2 \cdot \frac{3\pi}{7}\right) = \cos \frac{6\pi}{7}$$

$$26) \tan(2 \cdot 31^\circ) = \tan 62^\circ$$

$$27) \cos \frac{\pi/9}{2} = \cos \frac{\pi}{18}$$

$$28) \tan \frac{80^\circ}{2} = \tan 40^\circ$$

$$29) \sin 2x = \tan x (1 + \cos 2x)$$

$$\begin{aligned} \tan x (1 + \cos 2x) &= \tan x (1 + \cos^2 x - \sin^2 x) \\ &= \tan x (1 - \sin^2 x + \cos^2 x) \end{aligned}$$

$$= \tan x (\cos^2 x + \cos^2 x)$$

$$= \frac{\sin x}{\cos x} (2 \cos^2 x)$$

$$= \sin x \cdot 2 \cdot \cos x$$

$$= 2 \sin x \cos x = \sin 2x \quad \checkmark$$

$$30) \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{1 - \tan^2 x}{\sec^2 x} = \frac{1}{\sec^2 x} - \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$$

$$= \frac{1}{\sec^2 x} - \frac{\sin^2 x \cdot \frac{\cos^2 x}{1}}{\cos^2 x}$$

$$= \cos^2 x - \sin^2 x = \cos 2x \quad \checkmark$$