Set Theory

A set is a well-defined collection of objects. Upper case letters usually denote sets. (Recall some examples of sets: W – whole numbers, N – natural numbers, Z – integers, Q – rational numbers, R – real numbers, C – complex numbers)

The objects in a set are called elements or members. Lower case letters, such as a or b, usually denote elements. If x is an element of S then we write x $\in$ S.

ex: $3 \in W$ $\quad \frac{1}{2} \notin W$

Elements are usually enclosed in braces. N = {1, 2, 3, ...}. The series of dots (...) are called ellipses and mean "the pattern continues."

Sets can be written by rule in set-builder notation or by roster.

Example of set notations:

Set Builder $\rightarrow$ $S = \{x \mid x \in \mathbb{Z} \text{ and } 0 < x < 10\}$ $\quad 1 \le x \le 9$

Roster $\rightarrow$ $S = \{1, 2, 3, 4, \ldots, 9\}$

Two sets are equal iff they contain the same elements.

U usually stands for the universal set (the set containing all the members of some specific class, such as the set of all real numbers).

The complement of a set (written A' or Ā) contains everything in the universal set that is not in the set itself. $\sim A$

Example: Given: $\mathcal{U} = \{1, 2, 3, \ldots, 9\}$

and = $A = \{3, 4, 5\}$

$A' = \{1, 2, 6, 7, 8, 9\}$

i.e. $A \cup A' = \mathcal{U}$

Set A is a **subset** of B (written A B) iff every member of A is also in B. If A is a subset of B but not equal to B then A is a **proper subset** of B (written A B).

Example: $M = \{2,4,6,8\}$ $P = \{2,4,6\}$ $L = \{2,4,6,8\}$

$M = L$ $M \subseteq L$ $L \subseteq M$ $M \not\subset L$

$M \neq P$ $P \subseteq M$ $P \subset M$

If a set is **finite** (having a distinct number of elements) then the number of elements, n, in the set is called the **cardinality** and is written |S|. A set that is not finite is said to be **infinite**.

Example: $M = \{2,4,6,8\}$ $P = \{2,4,6\}$

$|M| = 4$ $|P| = 3$

A set with no elements is called the **null set** or **empty set** and is written ∅ or { }.

The **union** of A and B (written A ∪ B) is the set that contains all the elements of A and all the elements of B.

The **intersection** of A and B (written A ∩ B) is the set that contains the elements that are in both A and B. If the intersection of the two sets is the empty set, the sets are called **disjoint** sets.

The **difference set** of A and B (written A – B) is the set that contains all the elements in A that are not in B.

The difference of A and B is also called the complement of B with respect to A. The **symmetric difference** of A and B (written A ⊕ B) is the set that contains those elements in A or B, but not in both A and B.

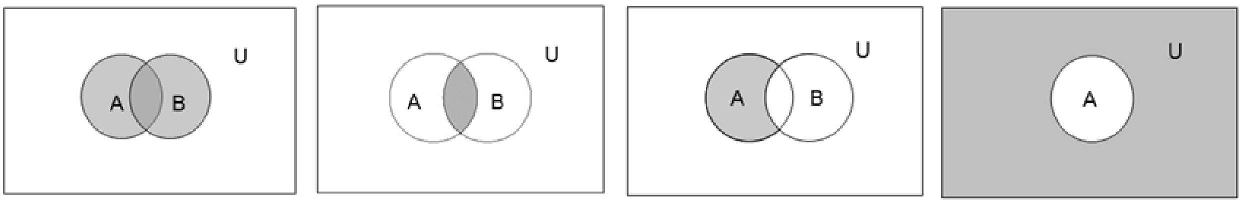
Example: Given: $A = \{1,2,3,4\}$ $B = \{2,4,6\}$ $C = \{5,6\}$

$A \cup B = \{1,2,3,4,6\}$
$A \cup C = \{1,2,3,\ldots,6\}$
$B \cup C = \{2,4,5,6\}$

$A \cap B = \{2,4\}$
$B \cap C = \{6\}$
$A \cap C = \{\ \}$
A and C are disjoint.

$A - B = \{1,3\}$
$B - A = \{6\}$
$A \oplus B = \{1,3,6\}$
$B \oplus C = \{2,4,5\}$

Venn diagrams, shown below, are used to illustrate sets and subsets.



$A \cup B$ $\quad$ $A \cap B$ $\quad$ $A - B$ $\quad$ A'

Sometimes the number of times an element occurs in a collection matters. **Multisets** are collections in which an element can occur as a member more than once. The number of times an element occurs is written in front of the element and is called its **multiplicity**. (For example, {3·a, 6·b} means "a" is an element 3 times and "b" is an element 6 times.)

If P and Q are multisets, **P ∪ Q** is the multiset where the multiplicity of each element is the maximum of its multiplicities in P and Q. **P ∩ Q** is the multiset where the multiplicity of each element is the minimum of its multiplicities in P and Q. **P – Q** is the multiset where the multiplicity of each element is the difference of its multiplicities in P and Q, *unless that number is negative, in which case its multiplicity is zero*. **P + Q** is the multiset where the multiplicity of each element is the sum of its multiplicities in P and Q.

Example: Given: $A=\{2\cdot a, 1\cdot b, 4\cdot c\}$ $B=\{3\cdot a, 2\cdot c, 1\cdot p\}$

$A \cup B=\{3\cdot a, 1\cdot b, 4\cdot c, 1\cdot p\}$

$A \cap B=\{2\cdot a, 2\cdot c\}$

$A - B=\{1\cdot b, 2\cdot c\}$ $\quad$ $B-A=\{1\cdot a, 1\cdot p\}$

$A + B=\{5\cdot a, 1\cdot b, 6\cdot c, 1\cdot p\}$