Precalculus - Simplifying and Verifying with Sum/Difference Identities

#1-7. Simplify each of the following. Remember—first step is always to **EXPAND**

1. $\cos\left(\theta-\frac{3\pi}{2}\right)$ $= \cos\theta\cos\frac{3\pi}{2} + \sin\theta\sin\frac{3\pi}{2}$

$= (\cos\theta)(0) + (\sin\theta)(-1) = -\sin\theta$

2. $\tan(\theta+\pi)$ $= \dfrac{\tan\theta+\tan\pi}{1-\tan\theta\tan\pi} = \dfrac{\tan\theta}{1} = \tan\theta$

$\tan\pi = 0$

3. $\cos(\pi-\theta)+\sin\left(\frac{\pi}{2}+\theta\right)$

$= \cos\pi\cos\theta + \sin\pi\sin\theta + \sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta$

$= -1\cdot\cos\theta + 0\cdot\sin\theta + 1\cdot\cos\theta + 0\cdot\sin\theta = 0$

4. $\sin(\theta+\pi)+\cos\left(\theta-\frac{\pi}{2}\right)$

$= \sin\theta\cos\pi + \cos\theta\sin\pi + \cos\theta\cos\frac{\pi}{2} + \sin\theta\sin\frac{\pi}{2}$

$= \sin\theta\cdot(-1) + \cos\theta\cdot 0 + \cos\theta\cdot 0 + \sin\theta\cdot 1 = 0$

5. $\tan(\theta+\pi)-\tan(\pi-\theta)$ $= \dfrac{\tan\theta+\tan\pi}{1-\tan\theta\tan\pi} - \dfrac{\tan\pi-\tan\theta}{1+\tan\pi\tan\theta}$

$= \dfrac{2\tan\theta}{1} = 2\tan\theta$

6. $\sin\left(\theta+\frac{\pi}{4}\right)+\sin\left(\theta-\frac{\pi}{4}\right)$

$= \sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4} + \sin\theta\cos\frac{\pi}{4} - \cos\theta\sin\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}\sin\theta + \frac{\sqrt{2}}{2}\cos\theta + \frac{\sqrt{2}}{2}\sin\theta - \frac{\sqrt{2}}{2}\cos\theta = \sqrt{2}\sin\theta$

7. $\cos\left(\theta+\frac{\pi}{4}\right)-\cos\left(\theta-\frac{\pi}{4}\right)$

$= \cos\theta\cdot\cos\frac{\pi}{4} - \sin\theta\sin\frac{\pi}{4} - \left(\cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4}\right)$

$= -2\sin\theta\cdot\frac{\sqrt{2}}{2} = -\sqrt{2}\sin\theta$

#8-13. Verify the following identities.

8. $\cos(\pi-\theta)+\sin\left(\frac{\pi}{2}+\theta\right)=0$

$\cos\pi\cos\theta+\sin\pi\sin\theta+\sin\frac{\pi}{2}\cos\theta+\cos\frac{\pi}{2}\sin\theta$

$=-1\cdot\cos\theta+\cos\theta=0$ ✓

9. $\sin(\theta+\pi)+\cos\left(\theta-\frac{\pi}{2}\right)=0$

$\sin\theta\cos\pi+\cos\theta\sin\pi+\cos\theta\cos\frac{\pi}{2}+\sin\theta\sin\frac{\pi}{2}$

$=-\sin\theta+\sin\theta=0$ ✓

10. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

$\sin x\cos y+\cos x\sin y+\sin x\cos y-\cos x\sin y$

$=2\sin x\cos y$ ✓

11. $\cos(x+y)+\cos(x-y)=2\cos x\cos y$

$\cos x\cos y-\sin x\sin y+\cos x\cos y+\sin x\sin y$

$=2\cos x\cos y$ ✓

12. $\tan(x+\pi)-\tan(\pi-x)=2\tan x$

$\frac{\tan x+\tan\pi}{1-\tan x\tan\pi}+\frac{\tan\pi+\tan x}{1-\tan\pi\tan x}=\frac{2\tan x}{1}=2\tan x$

13. $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$

$\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}=\frac{1-\tan\theta}{1+\tan\theta}$ ✓