

Simplifying Trig Expressions

$$1. (\sin \theta) (\csc \theta)$$

$$= (\sin \theta) \cdot \frac{1}{\sin \theta} \quad \begin{array}{l} \text{reciprocal} \\ \text{sub.} \end{array}$$

$$= \frac{\sin \theta}{\sin \theta}$$

$$= \boxed{1}$$

$$2. \frac{1}{\csc x} \cdot \cot x$$

$$= \sin x \cdot \cot x$$

$$= \sin x \cdot \frac{\cos x}{\sin x}$$

$$= \boxed{\cos x}$$

$$3. \cos^2 x \cdot \tan^2 x + \cos^2 x$$

$$= \cos^2 x (\tan^2 x + 1) \quad \begin{array}{l} c^2 \cdot t^2 + c^2 \\ c^2(t^2 + 1) \end{array}$$

$$= \cos^2 x \cdot \sec^2 x \quad \begin{array}{l} \text{Pyth.} \\ \text{sub.} \end{array}$$

$$= \cos^2 x \cdot \frac{1}{\cos^2 x} \quad \begin{array}{l} \text{recip.} \\ \text{sub.} \end{array}$$

$$= \boxed{1}$$

$$\cos^2 x \cdot \tan^2 x + \cos^2 x$$

$$\cancel{\cos^2 x} \cdot \frac{\sin^2 x}{\cancel{\cos^2 x}} + \cos^2 x$$

$$\sin^2 x + \cos^2 x$$

$$= \boxed{1}$$

$$4. \cos \theta \cdot \cot \theta + \sin \theta$$

$$= \cos \theta \cdot \frac{\cos \theta}{\sin \theta} + \sin \theta$$

$$= \frac{\cos^2 \theta}{\sin \theta} + \frac{\sin \theta \cdot \sin \theta}{\sin \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta}$$

$$= \frac{1}{\sin \theta} = \boxed{\csc \theta}$$

$$\begin{aligned}
 5. \quad & \cos x (\cos x - \sec x) \\
 &= \cos x \left(\cos x - \frac{1}{\cos x} \right) \\
 &= \cos^2 x - 1 \\
 &= -\sin^2 x
 \end{aligned}$$

$$\begin{aligned}
 \frac{\sin^2 x}{-\sin^2 x} + \cos^2 x &= 1 - \sin^2 x \\
 \cos^2 x - 1 &= -\sin^2 x \\
 \cos^2 x - 1 &= -\sin^2 x
 \end{aligned}$$

$$6. \quad \frac{1 - \cos^2 x}{1 + \cos x} = \frac{(1 + \cos x)(1 - \cos x)}{(1 + \cos x)} = \boxed{1 - \cos x}$$

$$\begin{aligned}
 \frac{1 - \cos^2 x}{1 + \cos x} &= \frac{\sin^2 x}{1 + \cos x} \cdot \frac{(1 - \cos x)}{(1 - \cos x)} = \frac{\sin^2 x (1 - \cos x)}{(1 - \cos^2 x)} \\
 &= \frac{\sin^2 x (1 - \cos x)}{\cancel{\sin^2 x}} = \boxed{1 - \cos x}
 \end{aligned}$$

$$7. \quad \cot^4 x + 2\cot^2 x + 1$$

$$\begin{aligned}
 & \left(\cot^2 x + 1 \right) \left(\cot^2 x + 1 \right) \\
 &= (\csc^2 x) \cdot (\csc^2 x) \\
 &= \boxed{\csc^4 x}
 \end{aligned}$$

$$8. \quad \frac{\sin(-x)}{\tan x}$$

$$\begin{aligned}
 &= \frac{-\sin x}{\tan x} \\
 &= \frac{-\sin x}{\frac{\sin x}{\cos x}} \\
 &= -\sin x \cdot \frac{\cos x}{\sin x} \\
 &= \boxed{-\cos x}
 \end{aligned}$$