Simplifying Trig Expressions

1. $(\sin\theta)(\csc\theta)$

reciprocal sub.

$= (\sin\theta) \cdot \dfrac{1}{\sin\theta}$

$= \dfrac{\sin\theta}{\sin\theta}$

$= \boxed{1}$

2. $\dfrac{1}{\csc x} \cdot \cot x$

$= \sin x \cdot \cot x$

$= \sin x \cdot \dfrac{\cos x}{\sin x}$

$= \boxed{\cos x}$

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3. $\cos^2 x \cdot \tan^2 x + \cos^2 x$

$c^2 t^2 + c^2$
$c^2(t^2+1)$

$\to = \cos^2 x (\tan^2 x + 1)$

$= \cos^2 x \cdot \sec^2 x$ Pyth. Sub.

$= \cos^2 x \cdot \dfrac{1}{\cos^2 x}$ recip. Sub.

$= \boxed{1}$

$\cos^2 x \cdot \tan^2 x + \cos^2 x$

$\cos^2 x \cdot \dfrac{\sin^2 x}{\cos^2 x} + \cos^2 x$

$\sin^2 x + \cos^2 x$

$= \boxed{1}$

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4. $\cos\theta \cdot \cot\theta + \sin\theta$

$= \cos\theta \cdot \dfrac{\cos\theta}{\sin\theta} + \sin\theta$

$= \dfrac{\cos^2\theta}{\sin\theta} + \dfrac{\sin\theta}{1} \cdot \dfrac{\sin\theta}{\sin\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} = \boxed{\csc\theta}$

5. $\cos x(\cos x - \sec x)$

$= \cos x \left(\cos x - \dfrac{1}{\cos x}\right)$

$= \cos^2 x - 1$

$= -\sin^2 x$

$\sin^2 x + \cos^2 x = 1$
$-\sin^2 x \qquad\qquad -\sin^2 x$

$\cos^2 x - 1 = 1 - \sin^2 x$
$\qquad\qquad -1$

$\cos^2 x - 1 = -\sin^2 x$

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6. $\dfrac{1 - \cos^2 x}{1 + \cos x} = \dfrac{(1 + \cos x)(1 - \cos x)}{(1 + \cos x)} = \boxed{1 - \cos x}$

$\dfrac{1 - c^2}{(1+c)(1-c)}$

$\dfrac{1 - \cos^2 x}{1 + \cos x} = \dfrac{\sin^2 x}{(1 + \cos x)} \cdot \dfrac{(1 - \cos x)}{(1 - \cos x)} = \dfrac{\sin^2 x (1 - \cos x)}{(1 - \cos^2 x)}$

$1 - \cos x + \cos x - \cos^2 x$

$= \dfrac{\sin^2 x (1 - \cos x)}{\sin^2 x} = \boxed{1 - \cos x}$

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7. $\cot^4 x + 2\cot^2 x + 1$

$c^4 + 2c^2 + 1$

$(c^2 + 1)(c^2 + 1)$
c^2
c^2

$= (\cot^2 x + 1)(\cot^2 x + 1)$

$= (\csc^2 x) \cdot (\csc^2 x)$

$= \boxed{\csc^4 x}$

8. $\dfrac{\sin(-x)}{\tan x}$

$= \dfrac{-\sin x}{\tan x}$

$= \dfrac{-\sin x}{\dfrac{\sin x}{\cos x}}$

$= \dfrac{-1}{\sin x} \cdot \dfrac{\cos x}{\sin x}$

$= \boxed{-\cos x}$