Simplifying, Verifying, Solving WS

1) $\cos 4\theta \cos(-6\theta) - \sin 4\theta \sin(-6\theta)$

$= \cos(4\theta + -6\theta) = \cos(-2\theta) = \boxed{\cos 2\theta}$

2) $\sin 6u \cdot \cos 5u + \cos 6u \cdot \sin 5u$

$= \sin(6u + 5u) = \boxed{\sin 11u}$

3) $\dfrac{\tan 2v + \tan v}{1 - \tan 2v \tan v} = \tan(2v + v) = \boxed{\tan 3v}$

4) $\dfrac{\tan 3u - \tan 5u}{1 + \tan 3u \tan 5u} = \tan(3u - 5u) = \tan(-2u)$

$= \boxed{-\tan 2u}$

5) $\sin 4\theta \cos 6\theta - \cos 4\theta \sin 6\theta$

$= \sin(4\theta - 6\theta) = \sin(-2\theta) = \boxed{-\sin 2\theta}$

6) $\cos(-3v) \cos 2v + \sin(-3v) \sin 2v$

$= \cos(-3v - 2v) = \cos(-5v) = \boxed{\cos 5v}$

7) $\sin \frac{5\pi}{18} \cos \frac{\pi}{9} - \cos \frac{5\pi}{18} \sin \frac{\pi}{9}$

$= \sin\left(\frac{5\pi}{18} - \frac{\pi}{9}\right) = \sin\left(\frac{5\pi}{18} - \frac{2\pi}{9}\right) = \sin\left(\frac{3\pi}{18}\right)$

$\sin \frac{3\pi}{18} = \sin \frac{\pi}{6} = \boxed{\frac{1}{2}}$

8) $\cos \frac{11\pi}{9} \cos \frac{17\pi}{36} + \sin \frac{11\pi}{9} \sin \frac{17\pi}{36}$

$= \cos\left(\frac{11\pi}{9} - \frac{17\pi}{36}\right) = \cos\left(\frac{44\pi}{36} - \frac{17\pi}{36}\right) = \cos \frac{27\pi}{36}$

$= \cos \frac{3\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$

9) $\dfrac{\tan\frac{17\pi}{9}-\tan\frac{5\pi}{9}}{1+\tan\frac{17\pi}{9}\tan\frac{5\pi}{9}} = \tan\left(\frac{17\pi}{9}-\frac{5\pi}{9}\right) = \tan\frac{12\pi}{9}$

$= \tan\frac{4\pi}{3} = \boxed{\sqrt{3}}$

10) $\dfrac{\tan\frac{\pi}{9}+\tan\frac{5\pi}{36}}{1-\tan\frac{\pi}{9}\tan\frac{5\pi}{36}} = \tan\left(\frac{\pi}{9}+\frac{5\pi}{36}\right) = \tan\left(\frac{4\pi}{36}+\frac{5\pi}{36}\right)$

$= \tan\frac{9\pi}{36} = \tan\frac{\pi}{4} = \boxed{1}$

11) $\cos\frac{13\pi}{18}\cos\frac{5\pi}{18} - \sin\frac{13\pi}{18}\sin\frac{5\pi}{18} = \cos\left(\frac{13\pi}{18}+\frac{5\pi}{18}\right)$

$= \cos\left(\frac{18\pi}{18}\right) = \cos\pi = \boxed{-1}$

12) $\sin\frac{2\pi}{9}\cos\frac{29\pi}{18} + \cos\frac{2\pi}{9}\sin\frac{29\pi}{18}$

$= \sin\left(\frac{2\pi}{9}+\frac{29\pi}{18}\right) = \sin\left(\frac{4\pi}{18}+\frac{29\pi}{18}\right)$

$= \sin\frac{33\pi}{18} = \sin\frac{11\pi}{6} = \boxed{\frac{-1}{2}}$

13) $\cos\left(\frac{3\pi}{2}-x\right) = \cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x$

$= 0\cdot\cos x + (-1)\cdot\sin x = \boxed{-\sin x}$

14) $\tan(x+\pi) = \dfrac{\tan x+\tan\pi}{1-\tan x\tan\pi} = \dfrac{\tan x+0}{1-0\cdot\tan x} = \boxed{\tan x}$

15) $\sin\left(\frac{3\pi}{2}+x\right) = \sin\frac{3\pi}{2}\cos x + \cos\frac{3\pi}{2}\sin x$

$= (-1)\cos x + 0\cdot\sin x = \boxed{-\cos x}$

16) $\sin\left(x-\frac{\pi}{2}\right) = \sin x\cos\frac{\pi}{2} - \cos x\sin\frac{\pi}{2}$

$= \sin x\cdot 0 - \cos x\cdot 1 = \boxed{-\cos x}$

17) $\tan\left(\frac{\pi}{4}-x\right) = \dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x} = \boxed{\dfrac{1-\tan x}{1+\tan x}}$

18) $\cos\left(x-\frac{\pi}{2}\right) = \cos x\cos\frac{\pi}{2} + \sin x\sin\frac{\pi}{2}$

$= \cos x\cdot 0 + \sin x\cdot 1 = \boxed{\sin x}$

19) $\sin\left(x+\frac{\pi}{6}\right) - \sin\left(x-\frac{\pi}{6}\right) = \frac{1}{2}$

$\sin x\cos\frac{\pi}{6} + \cos x\sin\frac{\pi}{6} - \left(\sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6}\right) = \frac{1}{2}$

$\cancel{\sin x\cos\frac{\pi}{6}} + \cos x\sin\frac{\pi}{6} - \cancel{\sin x\cos\frac{\pi}{6}} + \cos x\sin\frac{\pi}{6} = \frac{1}{2}$

$2\cos x\sin\frac{\pi}{6} = \frac{1}{2}$

$2\cdot\cos x\cdot\frac{1}{2} = \frac{1}{2} \nearrow$ $\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

20) $\sin\left(x+\frac{\pi}{2}\right) - \cos\left(x+\frac{3\pi}{2}\right) = 0$

$\sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2} - \left(\cos x\cos\frac{3\pi}{2} - \sin x\sin\frac{3\pi}{2}\right) = 0$

$\cancel{\sin x\cdot 0} + \cos x\cdot 1 - \cancel{\cos x\cdot 0} + \sin x\cdot(-1) = 0$

$\cos x - \sin x = 0$

$\cos x = \sin x$

$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

21) $\tan(x+\pi) + 2\sin(x+\pi) = 0$

$$\frac{\tan x + \cancel{\tan \pi}}{1 - \cancel{\tan x \tan \pi}} + 2(\sin x \cos \pi + \cos x \sin \pi) = 0$$

$$\tan x + 2\sin x \cdot (-1) + \cancel{2 \cdot \cos x \cdot 0} = 0$$

$$\tan x - 2\sin x = 0$$

$$\frac{\sin x}{\cos x} - 2\sin x = 0 \quad \leftarrow \text{multiply equation by } \cos x$$

$$\sin x - 2\sin x \cos x = 0$$

$$\sin x\,(1 - 2\cos x) = 0$$

$$\sin x = 0 \qquad \cos x = \frac{1}{2}$$

$$\boxed{x = 0\pi,\ \pi,\ \frac{\pi}{3},\ \frac{5\pi}{3}}$$

22) $2\sin\left(x + \frac{\pi}{2}\right) = \tan\frac{\pi}{3}$

$$2\left(\sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2}\right) = \tan\frac{\pi}{3}$$

$$\cancel{2\sin x \cdot 0} + 2\cos x \cdot 1 = \sqrt{3}$$

$$2\cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{\pi}{6},\ \frac{11\pi}{6}}$$