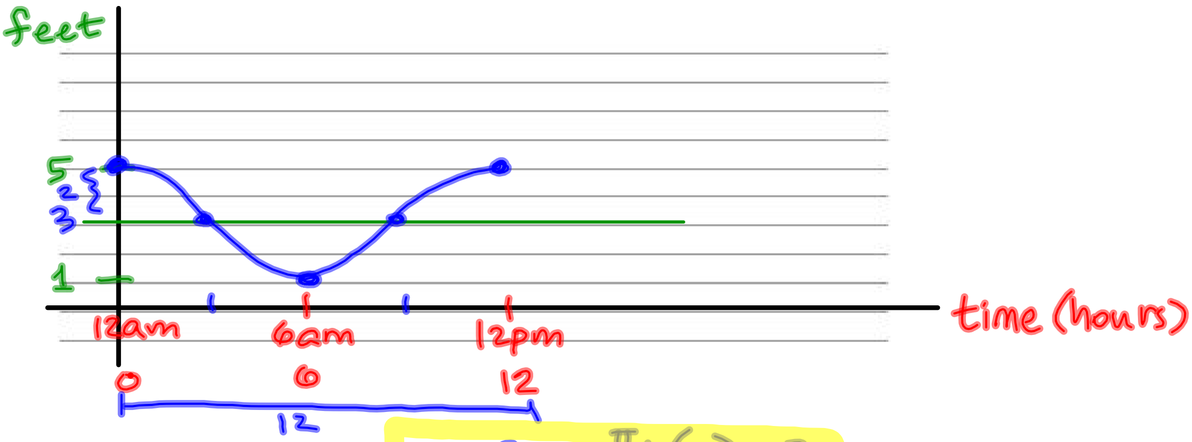
Sinusoidal Functions as Mathematical Models #1-4 solutions

1. *Tide Problem.* At high tide the ocean generally reaches the 5-foot mark on a retaining wall. At low tide the water reaches the 1-foot mark. Assume that high tide occurs at 12 noon and at midnight, and that low tide occurs at 6 pm and 6 am.

(a) Write a function to show how the height of the water varies over time.
(b) Determine the height of the water at 2:30 pm.
(c) Determine the first time after 6 am that the tide will reach the 4 foot mark.



$y = 2\cos\frac{\pi}{6}(x) + 3$

$\frac{2\pi}{12}$

D: [0,12]

R: [1,5]

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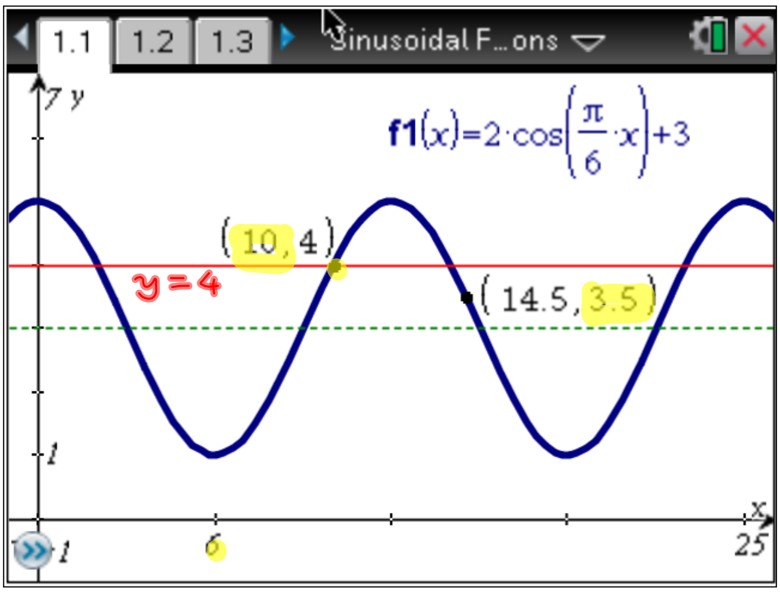
(a) Write a function to show how the height of the water varies over time.
(b) Determine the height of the water at 2:30 pm.

$\longrightarrow x = 14.5$ height = 3.5 feet

(c) Determine the first time after 6 am that the tide will reach the 4 foot mark.

10am



WINDOW:
xmin: −1
xmax: 25
xscl: 6
ymin: −1
ymax: 7
yscl: 1

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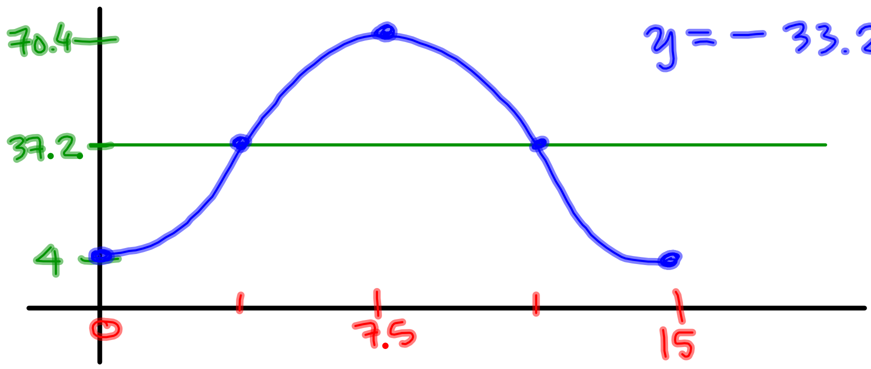
amp diam = 66.4

2. ***Ferris Wheel Problem.*** There is a ferris wheel every year at the North Georgia Fair. This year the wheel has a radius of 33.2 feet and makes a complete revolution every 15 seconds. For clearance, the bottom of the ferris wheel is 4 feet above the ground.

(a) Write a function to show how one passenger's height above the ground varies with time as he rides the ferris wheel.
(b) Determine how far above ground the passenger is 52 seconds into the ride.
(c) Determine the number of seconds the passenger has been on the ride when he first reaches a height of 27 feet.



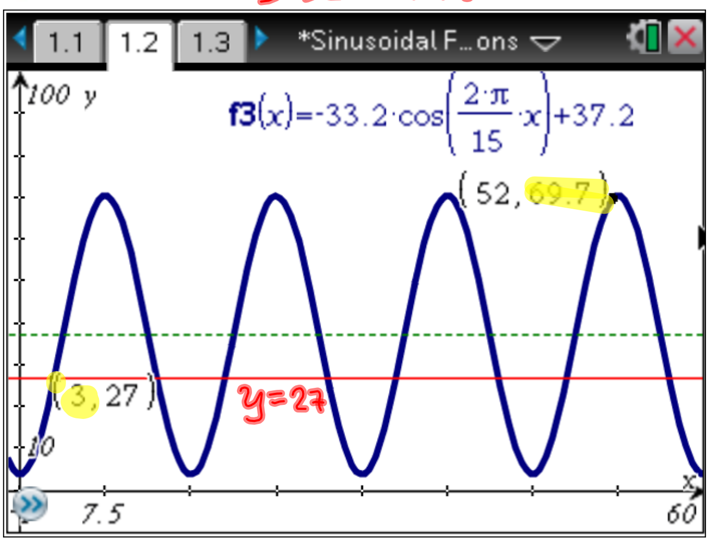
$y = -33.2\cos\frac{2\pi}{15}(x) + 37.2$

$\frac{2\pi}{15}$

D: [0, 15]
R: [4, 70.4]

2. ***Ferris Wheel Problem.*** There is a ferris wheel every year at the North Georgia Fair. This year the wheel has a radius of 33.2 feet and makes a complete revolution every 15 seconds. For clearance, the bottom of the ferris wheel is 4 feet above the ground.

(a) Write a function to show how one passenger's height above the ground varies with time as he rides the ferris wheel.
(b) Determine how far above ground the passenger is 52 seconds into the ride. 69.7 feet
(c) Determine the number of seconds the passenger has been on the ride when he first reaches a height of 27 feet. 3 seconds



WINDOW:
xmin: -1
xmax: 60
xscl: 7.5
ymin: -10
ymax: 100
yscl: 10

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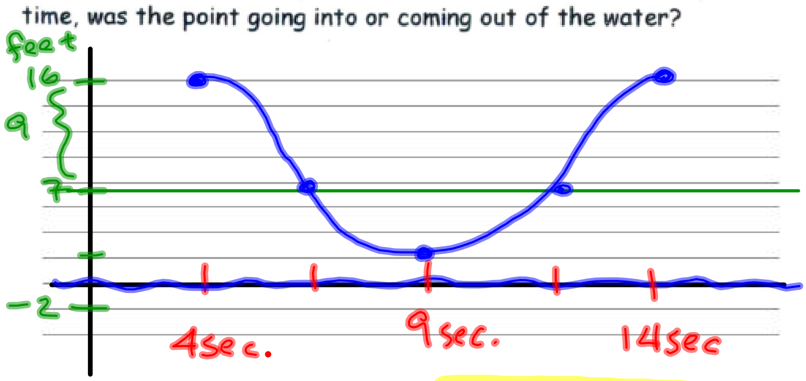
3. ***Steamboat Problem.*** Mark Twain sat on the deck of a river steamboat. As the paddle wheel turned, a point on the paddle blade moved so that its distance, d, from the water's surface was a sinusoidal function of time. When Twain's stopwatch read 4 seconds, the point was at its highest, 16 feet above the water's surface. The wheel's diameter was 18 feet, and it completed a revolution every 10 seconds.

(a) Sketch the graph of the sinusoid and write a function to represent your graph.
(b) What is the lowest the point goes? Why is it reasonable for this value to be negative?
(c) How far above the surface was the point when Mark's stopwatch read 17 seconds?
(d) What is the FIRST positive value of time at which the point was at the water's surface? At that time, was the point going into or coming out of the water?



$y = 9\cos\frac{\pi}{5}(x-4)+7$

$\frac{2\pi}{10}$

D: [4, 14]
R: [-2, 16]

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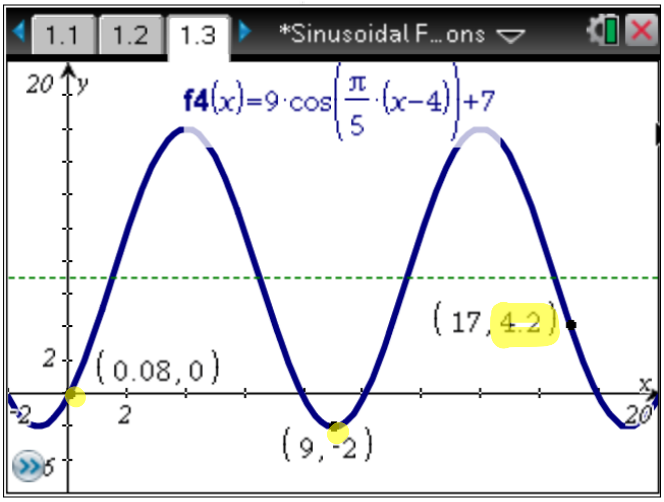
(a) Sketch the graph of the sinusoid and write a function to represent your graph.
(b) What is the lowest the point goes? Why is it reasonable for this value to be negative? -2 ft.
(c) How far above the surface was the point when Mark's stopwatch read 17 seconds? 4.2 ft.
(d) What is the FIRST positive value of time at which the point was at the water's surface? At that time, was the point going into or coming out of the water? 0.08 seconds

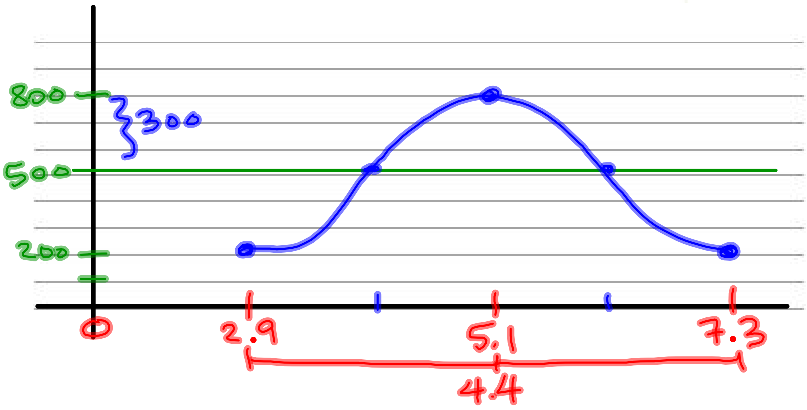


WINDOW
xmin: -2
xmax: 20
xscl: 2
ymin: -8
ymax: 20
yscl: 2

Sinusoidal Functions as Mathematical Models #1-4 solutions

4. ***Fox Population Problem.*** Naturalists find that populations of some kinds of predatory animals vary periodically with time. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept at time $t = 0$ *year*. A minimum number of 200 foxes appeared at $t = 2.9$ years. The next maximum, 800 foxes, occurred at $t = 5.1$ years.

(a) Sketch the graph of this sinusoid and write an equation expressing the number of foxes as a function of time.
(b) Predict the fox population when $t = 7, 8, 9,$ *and* 10 years.
(c) Foxes are declared an endangered species when their population drops below 300. Between what two nonnegative values of t did the foxes first become endangered?



$$y = -300\cos\frac{\pi}{2.2}(x-2.9)+500$$

$$\frac{2\pi}{4.4}$$

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