Sinusoidal Functions as Mathematical Models WS Name ______________________________

1. ***Tide Problem.*** At high tide the ocean generally reaches the 5-foot mark on a retaining wall. At low tide the water reaches the 1-foot mark. Assume that high tide occurs at 12 noon and at midnight, and that low tide occurs at 6 pm and 6 am.

(a) Graph and write a function to show how the height of the water varies over time.
(b) Determine the height of the water at 2:30 pm.
(c) Determine the first time after 6 am that the tide will reach the 4 foot mark.

2. ***Ferris Wheel Problem.*** There is a ferris wheel every year at the North Georgia Fair. This year the wheel has a radius of 33.2 feet and makes a complete revolution every 15 seconds. For clearance, the bottom of the Ferris wheel is 4 feet above the ground.

(a) Graph and write a function to show how one passenger's height above the ground varies with time as he rides the Ferris wheel.
(b) Determine how far above ground the passenger is 52 seconds into the ride.
(c) Determine the number of seconds the passenger has been on the ride when he first reaches a height of 27 feet.

3. ***Steamboat Problem.*** Mark Twain sat on the deck of a river steamboat. As the paddle wheel turned, a point on the paddle blade moved so that its distance, d, from the water's surface was a sinusoidal function of time. When Twain's stopwatch read 4 seconds, the point was at its highest,
16 feet above the water's surface. The wheel's diameter was 18 feet, and it completed a revolution every 10 seconds.

(a) Graph and write a function to represent your graph.
(b) What is the lowest the point goes? Why is it reasonable for this value to be negative?
(c) How far above the surface was the point when Mark's stopwatch read 17 seconds?
(d) What is the FIRST positive value of time at which the point was at the water's surface? At that time, was the point going into or coming out of the water?

4. ***Fox Population Problem.*** Naturalists find that populations of some kinds of predatory animals vary periodically with time. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept at time $t = 0$ *year*. A minimum number of 200 foxes appeared at $t = 2.9$ *years*. The following maximum, 800 foxes, occurred at $t = 5.1$ years.

(a) Graph and write an equation expressing the number of foxes as a function of time.
(b) Predict the fox population when $t = 7, 8, 9,$ *and* 10 years.
(c) Foxes are declared an endangered species when their population drops below 300. Between what two nonnegative values of t did the foxes first become endangered?

5. ***Bouncing Spring Problem.*** A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 seconds.

(a) Graph and find an equation for the distance from the floor as a function of time.
(b) What is the distance from the floor when the stopwatch reads 17.2 seconds?
(c) What was the distance from the floor when you started the stopwatch?
(d) What is the first positive value of time when the weight is 59 cm above the floor?

6. ***Rope Swing Problem.*** Zoey is at summer camp. One day she is swinging on a rope tied to a tree branch, going back and forth alternately over land and water. Nathan starts a stopwatch. When $x = 2$ seconds, Zoey is at one end of her swing, at a distance $y = -23$ feet from the riverbank. When $x = 5$ seconds, she is at the other end of her swing, at a distance $y = 17$ feet from the riverbank. Assume that while she is swinging, y varies sinusoidally with x.

(a) Graph and write an equation to model this problem.
(b) Find y when $x = 13.2$ seconds. Was Zoey over land or over water at this time?
(c) Find the first positive time when Zoey was directly over the riverbank ($y = 0$).
(d) Zoey lets go of the rope and splashes into the water. What is the value of y for the end of the rope when it comes to rest? What part of the mathematical model tells you this?

7. ***Sunspot Problem.*** For several hundred years, astronomers have kept track of the number of solar flares, or "sunspots", that occur on the surface of the Sun. The number of sunspots in a given year varies periodically, from a minimum of about 10 per year to a maximum of about 110 per year. Between 1750 and 1948, there were exactly 18 complete cycles.

(a) What is the period of a sunspot cycle?
(b) Assume that the number of sunspots per year is a sinusoidal function of time and that a maximum occurred in 1750 and at the end of each period. Find an equation expressing the number of sunspots per year as a function of the year.
(c) How many sunspots will there be in the year 2020?
(d) What is the first year after 2020 in which there will be about 35 sunspots?
(e) What is the first year after 2020 in which there will be a maximum number of sunspots?

8. ***Another Tide Problem.*** Suppose that you are on the beach at Port Aransas, Texas, on August 2. At 2:00 pm, at high tide, you find that the depth of the water at the end of a pier is 1.5 meters. At 7:30 pm, at low tide, the depth of the water is 1.1 meters. Assume that the depth varies sinusoidally with time.

(a) Graph and write an equation expressing depth as a function of the time that has elapsed since 12:00 midnight at the beginning of August 2.
(b) Predict the depth of the water at 5:00 pm on August 3.
(c) At what time does the first low tide occur on August 3?
(d) What is the earliest time on August 3 that the water depth will be 1.27 meters?

9. ***Pebble in the Tire Problem.*** As you stop your car at a traffic light, a pebble becomes wedged between your tire treads. When you start moving again, the distance between the pebble and the pavement varies sinusoidally with the distance that you have gone. The period is the circumference of the tire. The diameter of your tire is 24 inches.

(a) Graph and write an equation of the function that has **NO phase shift**.
(b) What is the pebble's distance from the pavement when you have gone 15 inches?
(c) What are the first two distances you have gone when the pebble is 11 inches from the pavement?

10. ***Porpoising Problem.*** Assume that you are aboard a research submarine doing submerged training exercises in the Pacific Ocean. At time $t = 0$, you start porpoising (going alternately deeper and shallower). At time $t = 4$ minutes you are at your deepest depth, -1000 meters. At time $t = 9$ minutes, you next reach your shallowest depth, -200 meters. Assume that the depth varies sinusoidally with time.

(a) Graph and write an equation expressing your depth as a function of time.
(b) Your submarine can't communicate with ships on the surface when it is deeper than -300 meters. At time $t = 0$, could your submarine communicate? Explain your answer.
(c) Between what two nonnegative times is your submarine first unable to communicate?