Solving Systems of Equations Using Inverse Matrices

Step 1: Use matrix multiplication to multiply the matrices in the equation below.

$$\begin{bmatrix} 5 & -4 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$5x + -4y = 8$

$x + 2y = 6$

☆ **You get a system of Equations!**

Step 2:

Working backward from step 1, we can write a system of equations as a matrix equation.

Standard form!

$Ax + By = C$

$$\begin{cases} 2x - y = -10 \\ -3x + 4y = 5 \end{cases}$$

$$\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$$

Step 3: Now we can solve the matrix equation using an inverse matrix!

2x2 det = $8 - (3) = 5$

$$\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$$

Multiply both sides of the equation by the inverse of the 2x2 matrix:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -10 \\ 5 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -35 \\ -20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \end{bmatrix}$$

so … x = -7 and y = -4 !

$(-7, -4)$

Here are some for you to try …

Example 1: Solve the system of equations using an inverse matrix.

$$\begin{cases} 2x + 5y = 19 \\ 3x + 2y = 1 \end{cases}$$

1) standard form

2) matrix equation

$$\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 1 \end{bmatrix}$$

3) 2x2 inverse

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 19 \\ 1 \end{bmatrix}$$

4) multiply matrices

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11} \begin{bmatrix} 3 & 3 \\ -5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$

Example 2: Solve the system of equations using an inverse matrix.

$$5x + 7y = 9$$

$$\frac{3}{1}\left(y = -\frac{2}{3}x + 1\right) \rightarrow 3y = -2x + 3$$

$$2x + 3y = 3$$

$$\begin{cases} 5x + 7y = 9 \\ 2x + 3y = 3 \end{cases}$$

$$\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$15 - 14 = 1$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$$