

Solve over  $[0, 2\pi)$ .

<p>1. <math>\tan x = 2 \sin x</math></p> <p><math>\cos x \left( \frac{\sin x}{\cos x} = 2 \sin x \right)</math></p> <p><math>\sin x - 2 \sin x \cos x = 0</math></p> <p><math>\sin x (1 - 2 \cos x) = 0</math></p> <p><math>\sin x = 0 \quad \cos x = \frac{1}{2}</math></p> <p><math>x = 0\pi, \pi, \frac{\pi}{3}, \frac{5\pi}{3}</math></p>	<p>2. <math>1 + \sin x = 2(\cos^2 x)</math></p> <p><math>1 + \sin x = 2(1 - \sin^2 x)</math></p> <p><math>1 + \sin x = 2 - 2 \sin^2 x</math></p> <p><math>2 \sin^2 x + \sin x - 1 = 0</math></p> <p><math>(2 \sin x - 1)(\sin x + 1) = 0</math></p> <p><math>\sin x = \frac{1}{2} \quad \sin x = -1</math></p> <p><math>x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}</math></p>	<p>3. <math>(\sin^2 x) = 2 \cos x + 2</math></p> <p><math>1 - \cos^2 x = 2 \cos x + 2</math></p> <p><math>0 = \cos^2 x + 2 \cos x + 1</math></p> <p><math>(\cos x + 1)(\cos x + 1) = 0</math></p> <p><math>\cos x = -1</math></p> <p><math>x = \pi</math></p>
<p>4. <math>\tan x = \cot x</math></p> <p>use logic...</p> <p><math>\tan</math> &amp; <math>\cot</math> are the same at</p> <p><math>x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}</math></p>	<p>5. <math>\csc^2 x = \cot x + 1</math></p> <p><math>1 + \cot^2 x = \cot x + 1</math></p> <p><math>\cot^2 x - \cot x = 0</math></p> <p><math>\cot x (\cot x - 1) = 0</math></p> <p><math>\cot x = 0 \quad \cot x = 1</math></p> <p><math>x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}</math></p>	<p>6. <math>\tan^2 x = -\frac{3}{2} \sec x</math></p> <p><math>2(\sec^2 x - 1) = -\frac{3}{2} \sec x</math></p> <p><math>2 \sec^2 x - 2 = -\frac{3}{2} \sec x</math></p> <p><math>2 \sec^2 x + \frac{3}{2} \sec x - 2 = 0</math></p> <p><math>(2 \sec x - 1)(\sec x + 2) = 0</math></p> <p><math>\sec x = \frac{1}{2} \quad \sec x = -2</math></p> <p><del><math>\cos x = 2</math></del> <del><math>\cos x = -\frac{1}{2}</math></del></p> <p><math>x = \frac{2\pi}{3}, \frac{4\pi}{3}</math></p>
<p>7. <math>\sin x \tan x = -\tan x</math></p> <p><math>\sin x \tan x + \tan x = 0</math></p> <p><math>\tan x (\sin x + 1) = 0</math></p> <p><math>\tan x = 0 \quad \sin x = -1</math></p> <p><math>x = 0\pi, \pi</math></p>	<p>8. <math>2 \sin^2 x = 3 \sin x - 1</math></p> <p><math>2 \sin^2 x - 3 \sin x + 1 = 0</math></p> <p><math>(2 \sin x - 1)(\sin x - 1) = 0</math></p> <p><math>\sin x = \frac{1}{2} \quad \sin x = 1</math></p> <p><math>x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}</math></p>	<p>9. <math>2 \sin^2 x = \sqrt{3} \sin x</math></p> <p><math>2 \sin^2 x - \sqrt{3} \sin x = 0</math></p> <p><math>\sin x (2 \sin x - \sqrt{3}) = 0</math></p> <p><math>\sin x = 0 \quad \sin x = \frac{\sqrt{3}}{2}</math></p> <p><math>x = 0\pi, \pi, \frac{\pi}{3}, \frac{2\pi}{3}</math></p>
<p>10. <math>\cot^2 x + \csc^2 x = 3</math></p> <p><math>\cot^2 x + \cot^2 x + 1 = 3</math></p> <p><math>2 \cot^2 x = 2</math></p> <p><math>\cot^2 x = 1</math></p> <p><math>\cot x = \pm 1</math></p> <p><math>x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}</math></p>	<p>11. <math>2 \cos x \csc x = \sqrt{3} \csc x</math></p> <p><math>2 \cos x \csc x - \sqrt{3} \csc x = 0</math></p> <p><math>\csc x (2 \cos x - \sqrt{3}) = 0</math></p> <p><math>\csc x = 0 \quad \cos x = \frac{\sqrt{3}}{2}</math></p> <p><del><math>\sin x = \frac{1}{0}</math></del></p> <p><math>x = \frac{\pi}{6}, \frac{11\pi}{6}</math></p>	<p>12. <math>3 \cos x + 3 = 2(\sin^2 x)</math></p> <p><math>3 \cos x + 3 = 2(1 - \cos^2 x)</math></p> <p><math>3 \cos x + 3 = 2 - 2 \cos^2 x</math></p> <p><math>2 \cos^2 x + 3 \cos x + 1 = 0</math></p> <p><math>(2 \cos x + 1)(\cos x + 1) = 0</math></p> <p><math>\cos x = -\frac{1}{2} \quad \cos x = -1</math></p> <p><math>x = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi</math></p>

$$13. \tan^2 x = \sqrt{3} \tan x$$

$$\begin{aligned} \tan^2 x - \sqrt{3} \tan x &= 0 \\ \tan x (\tan x - \sqrt{3}) &= 0 \\ \tan x &= 0 \quad \tan x = \sqrt{3} \end{aligned}$$

$$x = 0\pi, \pi, \frac{\pi}{3}, \frac{4\pi}{3}$$

$$14. (\tan x - 1)(\sec x - 1) = 0$$

$$\tan x = 1 \quad \sec x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}, 0\pi$$

$$15. \sec^2 x - 2 \tan x = 0$$

$$\begin{aligned} \tan^2 x + 1 - 2 \tan x &= 0 \\ \tan^2 x - 2 \tan x + 1 &= 0 \\ (\tan x - 1)(\tan x - 1) &= 0 \\ \tan x &= 1 \end{aligned}$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$16. (\sin^2 x - 1)(\tan x + 1) = 0$$

$$\sin^2 x = 1 \quad \tan x = -1$$

$$\sin x = \pm 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$17. 3 \cos x + \sqrt{2} = \cos x$$

$$- \cos x \quad - \cos x$$

$$2 \cos x + \sqrt{2} = 0$$

$$\cos x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$18. (\sec^2 x - 2)(\csc x + 1) = 0$$

$$\sec^2 x = 2 \quad \csc x = -1$$

$$\cos x = \pm \frac{\sqrt{2}}{2} \quad \sin x = -1$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$19. \cot x (\csc x + 2) = 0$$

$$\cot x = 0 \quad \csc x = -2$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$20. 2 \cos^2 x - 7 \cos x = -3$$

$$2 \cos^2 x - 7 \cos x + 3 = 0$$

$$(2 \cos x - 1)(\cos x - 3) = 0$$

$$\cos x = \frac{1}{2} \quad \cos x = 3$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$21. 6 \sin 2x - 3 = 0$$

$$\sin 2x = \frac{1}{2}$$

$$\textcircled{1} 2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\textcircled{2} 2x = \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

$$22. \tan 3x (\tan x - 1) = 0$$

$$\tan 3x = 0 \quad \tan x = 1$$

$$\textcircled{1} 3x = 0\pi, \pi \quad x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\textcircled{2} 3x = 2\pi, 3\pi$$

$$\textcircled{3} 3x = 4\pi, 5\pi$$

$$x = 0\pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \pi, \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$23. 3 \tan^2 2x = 1$$

$$\tan^2 2x = \frac{1}{3}$$

$$\tan 2x = \pm \frac{\sqrt{3}}{3}$$

$$\textcircled{1} 2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\textcircled{2} 2x = \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$$

$$\frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

$$24. 4 \sec 3x + 8 = 0$$

$$\sec 3x = -2$$

$$\cos 3x = -\frac{1}{2}$$

$$\textcircled{1} 3x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\textcircled{2} 3x = \frac{8\pi}{3}, \frac{10\pi}{3}$$

$$\textcircled{3} 3x = \frac{14\pi}{3}, \frac{16\pi}{3}$$

$$x = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}$$