

Solve the following equations over $[0, 2\pi)$.

1. $\csc^2 x + 2 \csc x = 0$

$$\csc x (\csc x + 2) = 0$$

$$\csc x = 0 \quad \csc x = -2$$

$$\cancel{\sin x = \frac{1}{0}} \quad \sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

2. $\cos^2 x - \cos x - 2 = 0$

$$(\cos x - 2)(\cos x + 1) = 0$$

$$\cancel{\cos x = 2} \quad \cos x = -1$$

$$x = \pi$$

3. $2 \cos x \sin x = \cos x$

$$2 \cos x \sin x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0 \quad \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

4. $2 = \sec x + \sec^2 x$

$$\sec^2 x + \sec x - 2 = 0$$

$$(\sec x - 1)(\sec x + 2) = 0$$

$$\sec x = 1 \quad \sec x = -2$$

$$\cos x = 1 \quad \cos x = -\frac{1}{2}$$

$$x = 0\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$$

5. $2 \sin^2 x + 5 \sin x = 3$

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

$$(2 \sin x - 1)(\sin x + 3) = 0$$

$$\sin x = \frac{1}{2} \quad \cancel{\sin x = -3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

6. $\tan^2 x \sin x = \sin x$

$$\tan^2 x \sin x - \sin x = 0$$

$$\sin x (\tan^2 x - 1) = 0$$

$$\sin x = 0 \quad \tan^2 x = 1$$

$$\tan x = \pm 1$$

$$x = 0\pi, \pi$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

7. $\sin x + \sin x \tan x = 0$

$$\sin x (1 + \tan x) = 0$$

$$\sin x = 0 \quad \tan x = -1$$

$$x = 0\pi, \pi, \frac{3\pi}{4}, \frac{7\pi}{4}$$

8. $\sec x \sin x - 2 \sin x = 0$

$$\sin x (\sec x - 2) = 0$$

$$\sin x = 0 \quad \sec x = 2$$

$$\cos x = \frac{1}{2}$$

$$x = 0\pi, \pi$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

9. $\sin^2 x - \tan x \sin^2 x = 0$

$$\cos^2 x (1 - \cot x) = 0$$

$$\cos^2 x = 0 \quad \cot x = 1$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}$$

10. $2 \tan^2 x - \tan x - 6 = 0$

$$(2 \csc x - 2)(\csc x + 2) = 0$$

$$\csc x = \frac{2}{3} \quad \csc x = -2$$

$$\cancel{\sin x = \frac{3}{2}} \quad \sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

11. $4 \cos^4 x - 5 \cos^2 x + 1 = 0$

$$(4 \cos^2 x - 1)(\cos^2 x - 1) = 0$$

$$\cos^2 x = \frac{1}{4} \quad \cos^2 x = 1$$

$$\cos x = \pm \frac{1}{2} \quad \cos x = \pm 1$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, 0\pi, \pi$$

12. $4 \sin^4 x + \sin^2 x - 3 = 0$

$$(4 \sin^2 x - 3)(\sin^2 x + 1) = 0$$

$$\sin^2 x = \frac{3}{4} \quad \cancel{\sin^2 x = -1}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$