

Solving Trig Equations with Factoring WS
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Name Taylor

Solve the following equations over $[0, 2\pi)$.

1. $\csc^2 x + 2 \csc x = 0$

$$\begin{aligned} \csc x (\csc x + 2) &= 0 \\ \csc x &= 0 \quad \csc x = -2 \\ \sin x &\neq 0 \quad \sin x = -\frac{1}{2} \end{aligned}$$

2. $\cos^2 x - \cos x - 2 = 0$

$$\begin{aligned} (\cos x - 2)(\cos x + 1) &= 0 \\ \cos x &\neq 2 \quad \cos x = -1 \\ x &= \pi \end{aligned}$$

3. $2 \cos x \sin x = -\cos x$

$$\begin{aligned} 2 \cos x \sin x - \cos x &= 0 \\ \cos x (2 \sin x - 1) &= 0 \\ \cos x &= 0 \quad \sin x = \frac{1}{2} \\ x &= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \end{aligned}$$

5. $2 \sin^2 x + 5 \sin x = 3$

$$\begin{aligned} 2 \sin^2 x + 5 \sin x - 3 &= 0 \\ (2 \sin x - 1)(\sin x + 3) &= 0 \\ \sin x &= \frac{1}{2} \quad \sin x \neq -3 \\ x &= \frac{\pi}{6}, \frac{5\pi}{6} \end{aligned}$$

7. $\sin x + \sin x \tan x = 0$

$$\begin{aligned} \sin x (1 + \tan x) &= 0 \\ \sin x &= 0 \quad \tan x = -1 \\ x &= 0\pi, \pi, \frac{3\pi}{4}, \frac{7\pi}{4} \end{aligned}$$

9. $\sin^2 x - \tan x \sin^2 x = 0$

$$\begin{aligned} \sin^2 x (1 - \cot x) &= 0 \\ \sin^2 x &= 0 \quad \cot x = 1 \\ \sin x &= 0 \\ x &= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4} \end{aligned}$$

11. $4 \cos^4 x - 5 \cos^2 x + 1 = 0$

$$\begin{aligned} (4 \cos^2 x - 1)(\cos^2 x - 1) &= 0 \\ \cos^2 x &= \frac{1}{4} \quad \cos^2 x = 1 \\ \cos x &= \pm \frac{1}{2} \quad \cos x = \pm 1 \\ x &= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, 0\pi, \pi \end{aligned}$$

4. $2 = \sec x + \sec^2 x$

$$\begin{aligned} \sec^2 x + \sec x - 2 &= 0 \\ (\sec x - 1)(\sec x + 2) &= 0 \\ \sec x &= 1 \quad \sec x = -2 \\ \cos x &= 1 \quad \cos x = -\frac{1}{2} \\ x &= 0\pi, \frac{2\pi}{3}, \frac{4\pi}{3} \end{aligned}$$

6. $\tan^2 x \sin x = \sin x$

$$\begin{aligned} \tan^2 x \sin x - \sin x &= 0 \\ \sin x (\tan^2 x - 1) &= 0 \\ \sin x &= 0 \quad \tan^2 x = 1 \\ \sin x &= 0 \quad \tan x = \pm 1 \\ x &= 0\pi, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \end{aligned}$$

8. $\sec x \sin x - 2 \sin x = 0$

$$\begin{aligned} \sin x (\sec x - 2) &= 0 \\ \sin x &= 0 \quad \sec x = 2 \\ \sin x &= 0 \quad \cos x = \frac{1}{2} \\ x &= 0\pi, \pi, \frac{\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

10. $2 \tan^2 x - \tan x - 6 = 0$

$$\begin{aligned} (2 \csc x - 1)(\csc x + 2) &= 0 \\ \csc x &= \frac{1}{2} \quad \csc x = -2 \\ \sin x &\neq \frac{1}{2} \quad \sin x = -\frac{1}{2} \\ x &= \frac{7\pi}{6}, \frac{11\pi}{6} \end{aligned}$$

12. $4 \sin^4 x + \sin^2 x - 3 = 0$

$$\begin{aligned} (4 \sin^2 x - 3)(\sin^2 x + 1) &= 0 \\ \sin^2 x &= \frac{3}{4} \quad \sin^2 x = -1 \\ \sin x &= \pm \frac{\sqrt{3}}{2} \\ x &= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \end{aligned}$$