

# Solving Trig Equations

## FACTORIZING WITH PYTHAGOREAN SUBSTITUTIONS

Solve over  $[0, 2\pi)$ .

•  $\cos^2 x + \sin x = 1$

$$\begin{array}{c} \downarrow \\ 1 - \sin^2 x + \sin x = 1 \\ -1 \qquad \qquad \qquad -1 \end{array}$$

$$-\sin^2 x + \sin x = 0$$

$$-\sin x (\sin x - 1) = 0$$

$$-\sin x = 0 \qquad \sin x - 1 = 0$$

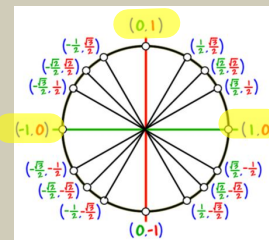
$$\sin x = 0 \qquad \sin x = 1$$

$$x = 0, \pi$$

$$x = \frac{\pi}{2}$$

$$\left\{ 0, \frac{\pi}{2}, \pi \right\}$$

$$\begin{array}{l} \sin^2 x + \cos^2 x = 1 \\ \cos^2 x = 1 - \sin^2 x \end{array}$$



Solve over  $[0, 2\pi)$ .

•  $\sin^2 x = \cos^2 x$

$$1 - \cancel{\cos^2 x} = \cos^2 x$$

$$+ \cos^2 x \quad + \cos^2 x$$

$$1 = 2 \cos^2 x$$

$$\frac{1}{2} = \cos^2 x$$

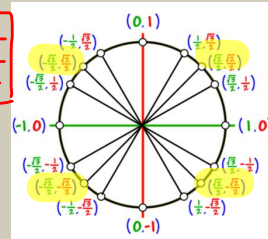
$$\pm \frac{1}{\sqrt{2}} = \cos x$$

$$\cos x = \pm \frac{\sqrt{2}}{2}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$



Solve over  $[0, 2\pi)$ .

•  $2\sin^2 x = 2 + \cos x$

$$2(1 - \cos^2 x) = 2 + \cos x$$

$$\cancel{2} - 2\cos^2 x = \cancel{2} + \cos x$$

$$-2\cos^2 x - \cos x = 0$$

$$2\cos^2 x + \cos x = 0$$

$$\cos x (2\cos x + 1) = 0$$

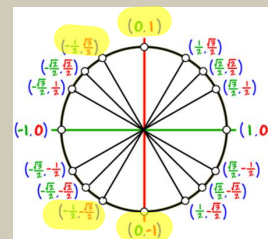
$$\cos x = 0 \quad 2\cos x + 1 = 0$$

$$\cos x = -\frac{1}{2}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$$



Solve over  $[0, 2\pi)$ .

•  $\csc^2 x = 2\cot x$

$$\cot^2 x + 1 = 2\cot x$$

$$\cot^2 x - 2\cot x + 1 = 0$$

$$(\cot x - 1)(\cot x - 1) = 0$$

$$\cot x - 1 = 0$$

$$\cot x = 1$$

$$\tan x = 1$$

$$x = \left[ \frac{\pi}{4}, \frac{5\pi}{4} \right]$$

$$\cot^2 x + 1 = \csc^2 x$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1)$$

