

Solve each of the following over $[0, 2\pi)$.

<p>1. $\sin\left(\frac{\pi}{2} - x\right) = \frac{1}{2}$ $\sin \frac{\pi}{2} \cos x - \cos \frac{\pi}{2} \sin x = \frac{1}{2}$</p> <p style="text-align: center;">$\cos x = \frac{1}{2}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$</p>
<p>2. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$</p> <p>$\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3} + \sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} = 1$</p> <p>$2 \sin x \cos \frac{\pi}{3} = 1 \rightarrow \sin x = 1$ $x = \frac{\pi}{2}$</p>
<p>3. $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$</p> <p>$\sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6} - (\sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6}) = \frac{1}{2}$</p> <p>$2 \cos x \sin \frac{\pi}{6} = \frac{1}{2} \rightarrow \cos x = \frac{1}{2}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$</p>
<p>4. $\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$</p> <p>$\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4} - (\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}) = 1$</p> <p>$-2 \sin x \sin \frac{\pi}{4} = 1 \rightarrow -\sin x = \frac{1}{\sqrt{2}} \rightarrow \sin x = -\frac{\sqrt{2}}{2}$ $x = \frac{5\pi}{4}, \frac{7\pi}{4}$</p>
<p>5. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$</p> <p>$\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6} - (\cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6}) = 1$</p> <p>$-2 \sin x \sin \frac{\pi}{6} = 1 \rightarrow -\sin x = \frac{1}{2} \rightarrow \sin x = -\frac{1}{2}$ $x = \frac{3\pi}{2}$</p>
<p>6. $\cos(x + 3\pi) = \cos x + \sqrt{3}$</p> <p>$\cos x \cos 3\pi - \sin x \sin 3\pi - \cos x = \sqrt{3}$</p> <p>$-\cos x - \cos x = \sqrt{3}$ $-2 \cos x = \sqrt{3} \rightarrow \cos x = -\frac{\sqrt{3}}{2}$ $x = \frac{5\pi}{6}, \frac{7\pi}{6}$</p>

$$7. \sin\left(x + \frac{7\pi}{2}\right) = \cos^2 x - 2$$

$$\cancel{\sin x \cos \frac{7\pi}{2}} + \cos x \cancel{\sin \frac{7\pi}{2}} = \cos^2 x - 2$$

$$\cos x = 1$$

$$\begin{aligned} \cos^2 x + \cos x - 2 &= 0 \\ (\cos x + 2)(\cos x - 1) &= 0 \end{aligned}$$

$$x = 0$$

$$8. \cos\left(x - \frac{\pi}{2}\right) + 4\sin x = 2\sin^2 x - 3$$

$$\cancel{\cos x \cos \frac{\pi}{2}} + \sin x \cancel{\sin \frac{\pi}{2}} + 4\sin x = 2\sin^2 x - 3$$

$$\sin x = -\frac{1}{2}$$

$$\begin{aligned} 2\sin^2 x - 5\sin x - 3 &= 0 \\ (2\sin x + 1)(\sin x - 3) &= 0 \end{aligned}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$9. \tan(x + 5\pi) = 2\tan x + 1$$

$$\frac{\tan x + \cancel{\tan 5\pi}}{1 - \cancel{\tan x \tan 5\pi}} = 2\tan x + 1$$

$$\tan x = -1$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$10. \cos\left(x - \frac{3\pi}{2}\right) + \cos^2 x = 6 + 5\sin x$$

$$\cancel{\cos x \cos \frac{3\pi}{2}} + \sin x \cancel{\sin \frac{3\pi}{2}} + \cos^2 x = 6 + 5\sin x$$

$$\sin x = -1$$

$$-\sin x + 1 - \sin^2 x = 6 + 5\sin x$$

$$\sin^2 x + 6\sin x + 5 = 0$$

$$(\sin x + 5)(\sin x + 1) = 0$$

$$x = \frac{3\pi}{2}$$

$$11. \sin(\pi + x) = 2\sin^2 x - \sin x - 1$$

$$\cancel{\sin \pi \cos x} + \cos \pi \cancel{\sin x} = 2\sin^2 x - \sin x - 1$$

$$0 = 2\sin^2 x - 1$$

$$\sin^2 x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$12. \tan(x + \pi) + 2\sin(x + \pi) = 0$$

$$\frac{\cancel{\tan x} + \cancel{\tan \pi}}{1 - \cancel{\tan x \tan \pi}} + 2(\cancel{\sin x \cos \pi} + \cancel{\cos x \sin \pi}) = 0$$

$$\sin x (1 - 2\cos x) = 0$$

$$\sin x = 0 \quad \cos x = \frac{1}{2}$$

$$\tan x - 2\sin x = 0$$

$$x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\sin x - 2\sin x \cos x = 0$$

ANSWERS: 1. $\frac{\pi}{3}, \frac{5\pi}{3}$ 2. $\frac{\pi}{2}$ 3. $\frac{\pi}{3}, \frac{5\pi}{3}$ 4. $\frac{5\pi}{4}, \frac{7\pi}{4}$ 5. $\frac{3\pi}{2}$ 6. $\frac{5\pi}{6}, \frac{7\pi}{6}$ 7. 0 8. $\frac{7\pi}{6}, \frac{11\pi}{6}$

9. $\frac{3\pi}{4}, \frac{7\pi}{4}$ 10. $\frac{3\pi}{2}$ 11. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ 12. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$