

SUM AND DIFFERENCE IDENTITIES FOR COSINE

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

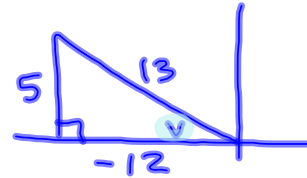
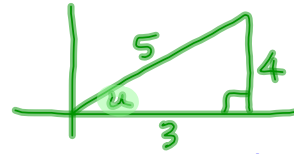
1. Use the sum or difference identities to find the **exact value**. \rightarrow no calculator answers.

$$\begin{aligned} & \cos 15^\circ \\ & = \cos(45^\circ - 30^\circ) \neq \cos 45^\circ - \cos 30^\circ \quad \text{NOT EQUAL!} \\ & = \cos(45^\circ)\cos(30^\circ) + \sin(45^\circ)\sin(30^\circ) \\ & = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ & = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\ & = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

2. Find the exact value of each trigonometric function, given:

$$\sin u = \frac{4}{5}, \text{ where } 0 < u < \frac{\pi}{2} \text{ and}$$

$$\cos v = -\frac{12}{13}, \text{ where } \frac{\pi}{2} < v < \pi.$$



a. $\cos(u + v)$

$$= \cos u \cos v - \sin u \sin v$$

$$= \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) = -\frac{36}{65} - \frac{20}{65} = \boxed{-\frac{56}{65}}$$

b. $\cos(v - u)$

$$= \cos v \cos u + \sin v \sin u$$

$$= \left(-\frac{12}{13}\right)\left(\frac{3}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = -\frac{36}{65} + \frac{20}{65} = \boxed{-\frac{16}{65}}$$