

## Sum & Difference Identities - Mixed w

10. a)  $\sin(15^\circ) = \sin(45^\circ - 30^\circ)$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

b)  $\cos(15^\circ) = \cos(45^\circ - 30^\circ)$

$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

c)  $\tan(15^\circ) = \tan(45^\circ - 30^\circ)$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}-1) \cdot (\sqrt{3}-1)}{(\sqrt{3}+1) \cdot (\sqrt{3}-1)} = \frac{4-2\sqrt{3}}{2}$$

16. a)  $\sin \frac{7\pi}{12} = \sin \left( \frac{5\pi}{4} + \frac{\pi}{6} \right)$

$$= \sin \frac{5\pi}{4} \cos \frac{\pi}{6} + \cos \frac{5\pi}{4} \sin \frac{\pi}{6}$$

$$= \left( -\frac{\sqrt{2}}{2} \right) \left( \frac{1}{2} \right) + \left( -\frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right) = \boxed{\frac{-\sqrt{2} - \sqrt{6}}{4}}$$

b)  $\cos \frac{7\pi}{12} = \cos \left( \frac{5\pi}{4} + \frac{\pi}{6} \right)$

$$= \cos \frac{5\pi}{4} \cos \frac{\pi}{6} - \sin \frac{5\pi}{4} \sin \frac{\pi}{6}$$

$$= \left( -\frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right) - \left( -\frac{\sqrt{2}}{2} \right) \left( \frac{1}{2} \right) = \frac{-\sqrt{6} + \sqrt{2}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

c)  $\tan \frac{7\pi}{12} = \tan \left( \frac{5\pi}{4} + \frac{\pi}{6} \right)$

$$= \frac{\tan \frac{5\pi}{4} + \tan \frac{\pi}{6}}{1 - \tan \frac{5\pi}{4} \tan \frac{\pi}{6}} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$= \frac{(\sqrt{3}+1) \cdot (\sqrt{3}+1)}{(\sqrt{3}-1) \cdot (\sqrt{3}+1)} = \frac{4+2\sqrt{3}}{2}$$

$$= \boxed{2 + \sqrt{3}}$$

$$\boxed{\cos \frac{12\pi}{5}}$$

$$19. \cos 40^\circ \cos 15^\circ - \sin 40^\circ \sin 15^\circ = \cos(40^\circ + 15^\circ) = \boxed{\cos 55^\circ}$$

$$20. \sin 110^\circ \cos 80^\circ + \cos 110^\circ \sin 80^\circ = \sin(110^\circ + 80^\circ) = \boxed{\sin 190^\circ}$$

$$21. \sin 340^\circ \cos 50^\circ - \cos 340^\circ \sin 50^\circ = \sin(340^\circ - 50^\circ) = \boxed{\sin 290^\circ}$$

$$22. \cos 20^\circ \cos 30^\circ + \sin 20^\circ \sin 30^\circ = \cos(30^\circ - 20^\circ) = \boxed{\cos 10^\circ}$$

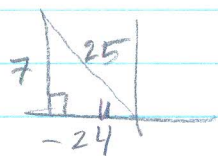
$$23. \frac{\tan 325^\circ - \tan 86^\circ}{1 - \tan 325^\circ \tan 86^\circ} = \tan(325^\circ - 86^\circ) = \boxed{\tan 239^\circ}$$

$$24. \frac{\tan 140^\circ - \tan 60^\circ}{1 + \tan 140^\circ \tan 60^\circ} = \tan(140^\circ - 60^\circ) = \boxed{\tan 80^\circ}$$

$$27. \cos \frac{\pi}{7} \cos \frac{\pi}{5} - \sin \frac{\pi}{7} \sin \frac{\pi}{5} = \cos\left(\frac{\pi}{7} + \frac{\pi}{5}\right)$$

$$28. \sin\left(\frac{2\pi}{9} + \frac{5\pi}{18}\right) = \sin \frac{29\pi}{18}$$

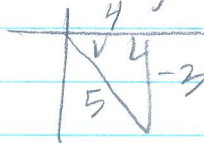
$$\sin u = \frac{7}{25} \text{ in QII}$$



$$\cos u = \frac{-24}{25}$$

$$\tan u = \frac{-7}{24}$$

$$\cos v = \frac{4}{5} \text{ in QIV}$$



$$\sin v = \frac{-3}{5}$$

$$\tan v = \frac{-3}{4}$$

$$39. \cos(u+v) = \left(\frac{-24}{25}\right)\left(\frac{4}{5}\right) - \left(\frac{7}{25}\right)\left(\frac{-3}{5}\right) = \frac{-96}{125} + \frac{21}{125} = \frac{-75}{125} = \boxed{\frac{-3}{5}}$$

$$40. \sin(u+v) = \left(\frac{7}{25}\right)\left(\frac{4}{5}\right) + \left(\frac{-24}{25}\right)\left(\frac{-3}{5}\right) = \frac{28}{125} + \frac{72}{125} = \frac{100}{125} = \boxed{\frac{4}{5}}$$

$$41. \sin(v-u) = \left(\frac{-3}{5}\right)\left(\frac{-24}{25}\right) - \left(\frac{4}{5}\right)\left(\frac{7}{25}\right) = \frac{72}{125} - \frac{28}{125} = \boxed{\frac{44}{125}}$$

$$42. \cos(u-v) = \left(\frac{-24}{25}\right)\left(\frac{4}{5}\right) + \left(\frac{7}{25}\right)\left(\frac{-3}{5}\right) = \frac{-96}{125} - \frac{21}{125} = \boxed{\frac{-117}{125}}$$

$$\bullet \tan(u+v) = \frac{\frac{-7}{24} + \frac{-3}{4}}{1 - \left(\frac{-7}{24}\right)\left(\frac{-3}{4}\right)} = \frac{\frac{-25}{24}}{\frac{75}{96}} = \boxed{\frac{-4}{3}}$$

$$\bullet \tan(u-v) = \frac{\frac{-7}{24} - \left(\frac{-3}{4}\right)}{1 + \left(\frac{-7}{24}\right)\left(\frac{-3}{4}\right)} = \frac{\frac{11}{24}}{\frac{117}{96}} = \boxed{\frac{44}{117}}$$