SUM AND DIFFERENCE IDENTITIES FOR TANGENT

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

1. Use the sum or difference identities to find the exact value.

$$\frac{17\pi}{12}=\frac{9\pi}{12}+\frac{8\pi}{12}=\frac{3\pi}{4}+\frac{2\pi}{3}$$

$$\tan\frac{17\pi}{12}=\tan\left(\frac{3\pi}{4}+\frac{2\pi}{3}\right)$$

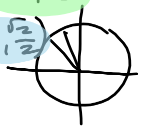
expansion $\longrightarrow$ $\dfrac{\tan\frac{3\pi}{4}+\tan\frac{2\pi}{3}}{1-\tan\frac{3\pi}{4}\cdot\tan\frac{2\pi}{3}}$

subst.

$$=\frac{(-1)+(-\sqrt{3})}{1-(-1)(-\sqrt{3})}$$

$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$



$$\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}=\frac{\sqrt{3}}{\cancel{2}}\cdot\left(\frac{-\cancel{2}}{1}\right)$$

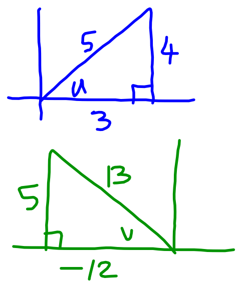
$$\frac{(-1-\sqrt{3})}{(1-\sqrt{3})}\cdot\frac{(1+\sqrt{3})}{(1+\sqrt{3})}=\frac{-1-\sqrt{3}-\sqrt{3}-3}{1+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}-3}$$

$$=\frac{-4-2\sqrt{3}}{-2}=\boxed{2+\sqrt{3}}$$

2. Find the exact value of each trigonometric function, given:

$\sin u = \frac{4}{5}$, where $0 < u < \frac{\pi}{2}$ and

$\cos v = -\frac{12}{13}$, where $\frac{\pi}{2} < v < \pi$.



a. $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \cdot \tan v} = \frac{\left(\frac{4}{3}\right) + \left(-\frac{5}{12}\right)}{1 - \left(\frac{4}{3}\right)\left(-\frac{5}{12}\right)}$

$$= \frac{\frac{4}{4}\cdot\frac{4}{3} - \frac{5}{12}}{\frac{36}{36}\cdot 1 + \frac{20}{36}} = \frac{\frac{16-5}{12}}{\frac{36+20}{36}} = \frac{\frac{11}{12}}{\frac{56}{36}}$$

$$= \frac{11}{\cancel{12}} \cdot \frac{\cancel{36}^{\,3}}{56} = \boxed{\frac{33}{56}}$$

~~b. tan(u − v)~~

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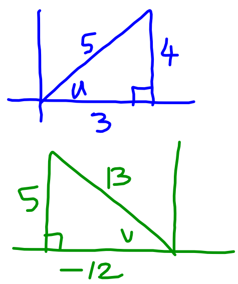

~~a. tan(u + v)~~

b. $\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \cdot \tan v} = \frac{\left(\frac{4}{3}\right) - \left(-\frac{5}{12}\right)}{1 + \left(\frac{4}{3}\right)\left(-\frac{5}{12}\right)}$

$$= \frac{\frac{4}{4}\cdot\frac{4}{3} + \frac{5}{12}}{\frac{36}{36}\cdot 1 - \frac{20}{36}} = \frac{\frac{21}{12}}{\frac{16}{36}}$$

$$= \frac{21}{\cancel{12}} \cdot \frac{\cancel{36}^{\,3}}{16} = \boxed{\frac{63}{16}}$$