

You can use the process of synthetic division/substitution for the following uses:

- dividing polynomials by a linear binomial ... answer is a full quotient + remainder
- evaluating a polynomial function ... answer is the value of the remainder
- factoring a polynomial function ... answer is a group of factors
- finding roots/zeros of a polynomial function ... answer is a solution set for "x"

Divide using synthetic division:

1. $(x^2 + 10) \div (x + 4)$

$$\begin{array}{r|rr} -4 & 1 & 0 & 10 \\ & & -4 & 16 \\ \hline & 1 & -4 & 26 \end{array}$$

$$\boxed{x - 4 + \frac{26}{x + 4}}$$

2. $(x^3 - 14x + 8) \div (x + 4)$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -14 & 8 \\ & & -4 & 16 & -8 \\ \hline & 1 & -4 & 2 & 0 \end{array}$$

$$\boxed{x^2 - 4x + 2}$$

3. $(2x^4 - 6x^3 + x^2 - 3x - 3) \div (x - 3)$

$$\begin{array}{r|rrrrr} 3 & 2 & -6 & 1 & -3 & -3 \\ & & 6 & 0 & 3 & 0 \\ \hline & 2 & 0 & 1 & 0 & -3 \end{array}$$

$$\boxed{2x^2 + x + \frac{-3}{x-3}}$$

4. $(x^3 - 8) \div (x - 2)$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

$$\boxed{x^2 + 2x + 4}$$

Evaluate each function using synthetic substitution.

5. $f(x) = 2x^3 - 3x^2 + 4x + 2$; $f(3)$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & 4 & 2 \\ & & 6 & 9 & 39 \\ \hline & 2 & 3 & 13 & 41 \end{array}$$

$$\boxed{f(3) = 41}$$

6. $f(x) = 2x^4 + 9x^3 + 4x^2 - 8x + 10$; $f(-3)$

$$\begin{array}{r|rrrrr} -3 & 2 & 9 & 4 & -8 & 10 \\ & & -6 & -9 & 15 & -21 \\ \hline & 2 & 3 & -5 & 7 & -11 \end{array}$$

$$\boxed{f(-3) = -11}$$

7. $f(x) = x^3 + 8x^2 + 8x - 32$; $f(-4)$

$$\begin{array}{r|rrrr} -4 & 1 & 8 & 8 & -32 \\ & & -4 & -16 & 32 \\ \hline & 1 & 4 & -8 & 0 \end{array}$$

$$\boxed{f(-4) = 0}$$

8. $f(x) = x^4 + 2x^3 - 63x^2 - 288x - 324$; $f(-4)$

$$\begin{array}{r|rrrrr} -4 & 1 & 2 & -63 & -288 & -324 \\ & & -4 & 8 & 220 & 272 \\ \hline & 1 & -2 & -55 & -68 & -52 \end{array}$$

$$\boxed{f(-4) = -52}$$

9. $f(x) = x^3 + 2x^2 - 5x - 6$; $f(1)$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -5 & -6 \\ & & 1 & 3 & -2 \\ \hline & 1 & 3 & -2 & -8 \end{array}$$

$$\boxed{f(1) = -8}$$

10. $f(x) = 12x^4 - 29x^3 - 60x^2 + 107x + 50$; $f(-2)$

$$\begin{array}{r|rrrrr} -2 & 12 & -29 & -60 & 107 & 50 \\ & & -24 & 106 & -92 & -30 \\ \hline & 12 & -53 & 46 & 15 & 20 \end{array}$$

$$\boxed{f(-2) = 20}$$

Factor each polynomial function, using the given information.

11. $f(x) = x^3 - 8x^2 + 5x + 14$; $f(2) = 0$

$$\begin{array}{r|rrrrr} 2 & 1 & -8 & 5 & 14 & \\ & & 2 & -12 & -14 & \\ \hline & 1 & -6 & -7 & 0 & \end{array}$$

$$x^2 - 6x - 7 \quad x=2$$

$$(x-7)(x+1)(x-2)$$

12. $f(x) = 12x^3 + 8x^2 - 13x + 3$; $x = \frac{1}{2}$ is one zero

$$\begin{array}{r|rrrrr} \frac{1}{2} & 12 & 8 & -13 & 3 & \\ & & 6 & 7 & -\frac{3}{2} & \\ \hline & 12 & 14 & -6 & 0 & \end{array}$$

$$12x^2 + 14x - 6 \quad x = \frac{1}{2}$$

$$2(6x^2 + 7x - 3)(x - \frac{1}{2})$$

13. $f(x) = x^3 + 3x^2 - 34x + 48$; $(x-3)$ is a factor

$$\begin{array}{r|rrrrr} 3 & 1 & 3 & -34 & 48 & \\ & & 3 & 18 & -48 & \\ \hline & 1 & 6 & -16 & 0 & \end{array}$$

$$(x^2 + 6x - 16)(x-3)$$

$$(x+8)(x-2)(x-3)$$

14. $f(x) = x^4 + 6x^3 - 4x^2 - 54x - 45$; $f(-5) = 0$

$$\begin{array}{r|rrrrrr} -5 & 1 & 6 & -4 & -54 & -45 & \\ & & -5 & -5 & 45 & 45 & \\ \hline & 1 & 1 & -9 & -9 & 0 & \end{array}$$

$$(x^3 + x^2 - 9x - 9)$$

$$x^2(x+1) - 9(x+1)$$

$$(x+1)(x^2 - 9)$$

$$(x+1)(x+3)(x-3)(x+5)$$

15. $f(x) = 2x^4 - 9x^3 + 4x^2 + 21x - 18$; $x = 2$ and $x = 3$ are roots

$$\begin{array}{r|rrrrrr} 2 & 2 & -9 & 4 & 21 & -18 & \\ & & 4 & -10 & -12 & 12 & \\ \hline & 2 & -5 & -6 & 9 & 0 & \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 2 & -5 & -6 & 9 & \\ & & 6 & 3 & -9 & \\ \hline & 2 & 1 & -3 & 0 & \end{array}$$

$$2x^2 + x - 3$$

$$(2x+3)(x-1)(x-2)(x-3)$$

Find the zeros of each function, using the given information.

16. $f(x) = x^3 + 2x^2 - 20x + 24$; $(x+6)$ is a factor

$$\begin{array}{r|rrrrr} -6 & 1 & 2 & -20 & 24 & \\ & & -6 & 24 & -24 & \\ \hline & 1 & -4 & 4 & 0 & \end{array}$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)(x-2) = 0$$

$$x = 2, -6$$

17. $f(x) = 2x^3 + 3x^2 - 3x - 2$; $f(-2) = 0$

$$\begin{array}{r|rrrrr} -2 & 2 & 3 & -3 & -2 & \\ & & -4 & 2 & 2 & \\ \hline & 2 & -1 & -1 & 0 & \end{array}$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$

$$x = -\frac{1}{2}, 1, -2$$

18. $f(x) = 2x^3 + 11x^2 + 9x + 2$; $(2x+1)$ is a factor

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & 11 & 9 & 2 & \\ & & -1 & -5 & -\frac{3}{2} & \\ \hline & 2 & 10 & 4 & 0 & \end{array}$$

$$2x^2 + 10x + 4 = 0$$

$$x^2 + 5x + 2 = 0$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

$$x = -\frac{1}{2}, \frac{-5 \pm \sqrt{17}}{2}$$

19. $f(x) = x^4 + 2x^3 - 14x^2 - 32x - 32$; ± 4 are zeros

$$\begin{array}{r|rrrrrr} 4 & 1 & 2 & -14 & -32 & -32 & \\ & & 4 & 24 & 40 & 32 & \\ \hline & 1 & 6 & 10 & 8 & 0 & \end{array}$$

$$\begin{array}{r|rrrrr} -4 & 1 & 6 & 10 & 8 & \\ & & -4 & -8 & -8 & \\ \hline & 1 & 2 & 2 & 0 & \end{array}$$

$$x^2 + 2x + 2 = 0$$

$$x^2 + 2x + 1 = -2 + 1$$

$$(x+1)^2 = -1$$

20. $f(x) = x^4 + 3x^3 + 7x^2 + 15x + 10$; $(x+2)$ is a factor

$$\begin{array}{r|rrrrr} -2 & 1 & 3 & 7 & 15 & 10 & \\ & & -2 & -2 & -10 & -10 & \\ \hline & 1 & 1 & 5 & 5 & 0 & \end{array}$$

$$x^3 + x^2 + 5x + 5 = 0$$

$$x^2(x+1) + 5(x+1) = 0$$

$$(x+1)(x^2 + 5) = 0$$

$$x = -1 \pm i, \pm 4$$

$$x^2 = -5$$

$$x = \pm i\sqrt{5}, 1, -2$$

** Challenge:

$f(x) = 3x^4 - 2x^3 - 12x^2 + 6x + 9$; $\pm\sqrt{3}$ are roots

Hint: start with $x = \sqrt{3}$ $x = -\sqrt{3}$

$$\text{so } (x - \sqrt{3})(x + \sqrt{3}) = x^2 - 3$$

now use long division and quadratic formula

$$x = \frac{1 \pm \sqrt{10}}{3}, \pm\sqrt{3}$$