Theoretical and Experimental Probability

Lesson Presentation

Objectives

- Solve problems involving the Fundamental Counting Principle.
- Find the theoretical probability of an event.
- Find the experimental probability of an event.

Vocabulary

Fundamental Counting Principle	
probability	outcome
sample space	event
equally likely outcomes	favorable outcomes
theoretical probability	complement
geometric probability	experiment
trial	experimental probability

You have previously used tree diagrams to find the number of possible combinations of a group of objects. In this lesson, you will learn to use the **Fundamental Counting Principle**.

Fundamental Counting Principle

If there are n items and m_1 ways to choose a first item, m_2 ways to choose a second item after the first item has been chosen, and so on, then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to choose n items.

Example 1: Using the Fundamental Counting Principle

To make a yogurt parfait, you choose one flavor of yogurt, one fruit topping, and one nut topping. How many parfait choices are there?

Yogurt Parfait (choose 1 of each)		
Flavor	**Fruit**	**Nuts**
Plain	Peaches	Almonds
Vanilla	Strawberries	Peanuts
	Bananas	Walnuts
	Raspberries	
	Blueberries	

Example 1 Continued

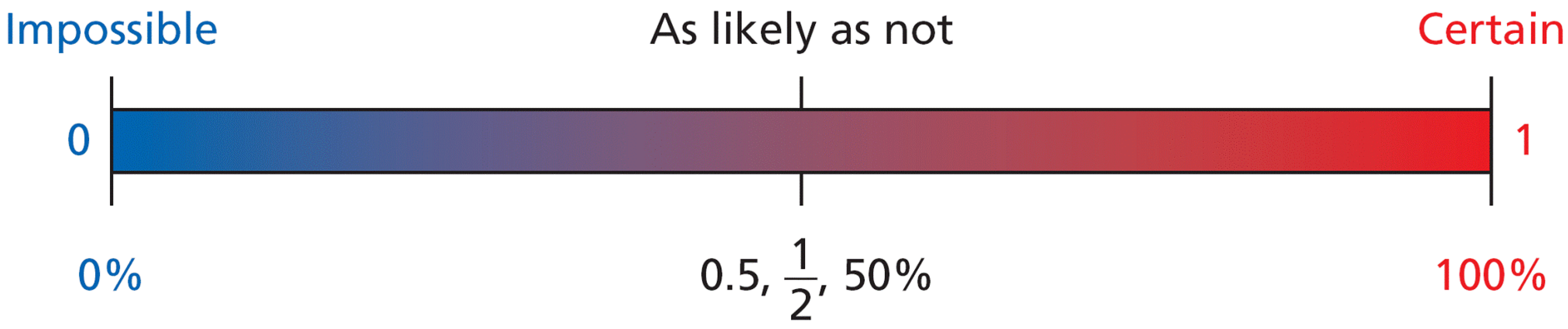
number of flavors	times	number of fruits	times	number of nuts	equals	number of choices
2	×	5	×	3	=	30

There are 30 parfait choices.

Probability is the measure of how likely an event is to occur. Each possible result of a probability experiment or situation is an **outcome**. The **sample space** is the set of all possible outcomes. An **event** is an outcome or set of outcomes.

Experiment or Situation	Rolling a number cube 	Spinning a spinner 
Sample Space	{1, 2, 3, 4, 5, 6}	{red, blue, green, yellow}

Probabilities are written as fractions or decimals from 0 to 1, or as percents from 0% to 100%.



Equally likely outcomes have the same chance of occurring. When you toss a fair coin, heads and tails are equally likely outcomes. **Favorable outcomes** are outcomes in a specified event. For equally likely outcomes, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the total number of outcomes.

Theoretical Probability

For equally likely outcomes,

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of outcomes in the sample space}}.$$

Example 2: Finding Theoretical Probability

Each letter of the word PROBABLE is written on a separate card. The cards are placed face down and mixed up. What is the probability that a randomly selected card has a consonant?

There are 8 possible outcomes and 5 favorable outcomes.

$$P(\text{consonant}) = \frac{5}{8} = 62.5\%$$

The sum of all probabilities in the sample space is 1. The **complement** of an event *E* is the set of all outcomes in the sample space that are not in *E*.

Complement

The probability of the complement of event *E* is

$$P(\text{not } E) = 1 - P(E).$$

Example 3: Application

There are 25 students in study hall. The table shows the number of students who are studying a foreign language. What is the probability that a randomly selected student is not studying a foreign language?

Language	Number
French	6
Spanish	12
Japanese	3

Example 3 Continued

$P(\text{not foreign}) = 1 - P(\text{foreign})$ *Use the complement.*

$$P(\text{not foreign}) = 1 - \frac{21}{25}$$

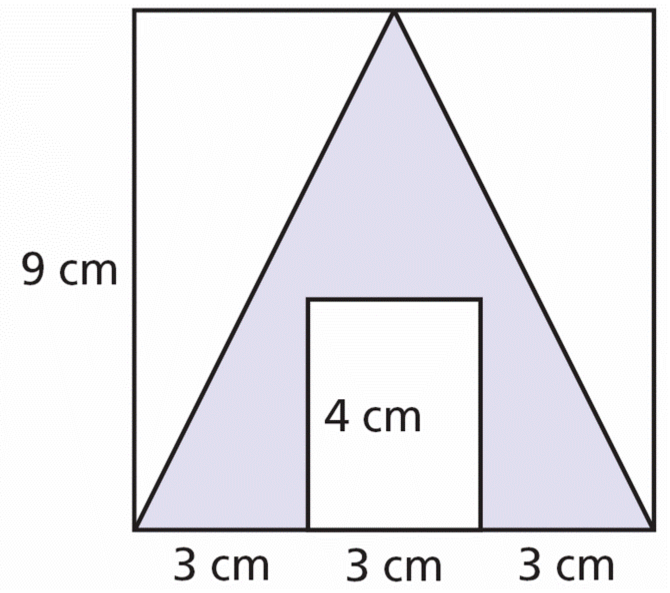
There are 21 students studying a foreign language.

$$= \frac{4}{25}\text{, or } 16\%$$

There is a 16% chance that the selected student is not studying a foreign language.

Example 4: Finding Geometric Probability

A figure is created placing a rectangle inside a triangle inside a square as shown. If a point inside the figure is chosen at random, what is the probability that the point is inside the shaded region?



Example 4 Continued

Find the ratio of the area of the shaded region to the area of the entire square. The area of a square is s^2, the area of a triangle is $\frac{1}{2}bh$, and the area of a rectangle is *lw*.

First, find the area of the entire square.

$A_t = (9)^2 = 81$ *Total area of the square.*

Example 4 Continued

Next, find the area of the triangle.

$$A_{\text{triangle}} = \frac{1}{2}(9)(9) = 40.5$$ *Area of the triangle.*

Next, find the area of the rectangle.

$$A_{\text{rectangle}} = (3)(4) = 12$$ *Area of the rectangle.*

Subtract to find the shaded area.

$$A_s = 40.5 - 12 = 28.5$$ *Area of the shaded region.*

$$\frac{A_s}{A_t} = \frac{28.5}{81} = \frac{19}{54} \approx 0.352$$ *Ratio of the shaded region to total area.*

You can estimate the probability of an event by using data, or by **experiment**. For example, if a doctor states that an operation "has an 80% probability of success," 80% is an estimate of probability based on similar case histories.

Each repetition of an experiment is a **trial**. The sample space of an experiment is the set of all possible outcomes. The **experimental probability** of an event is the ratio of the number of times that the event occurs, the *frequency*, to the number of trials.

Experimental Probability

$$\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Experimental probability is often used to estimate theoretical probability and to make predictions.

Example 5: Finding Experimental Probability

The table shows the results of a spinner experiment. Find the experimental probability.

Number	Occurrences
1	6
2	11
3	19
4	14

spinning a 4

The outcome of 4 occurred 14 times out of 50 trials.

$$P(4) = \frac{14}{50} = \frac{7}{25} = 0.28$$