UNIT VECTORS

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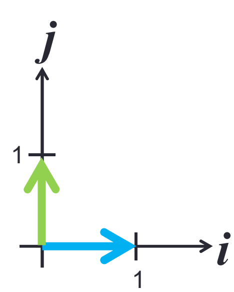
What is a UNIT VECTOR?

- A unit vector is a vector that is 1 unit long!

There are two standard unit vectors:

- $\vec{\imath}$ is a unit vector … $\vec{\imath} = \langle 1,0 \rangle$.
- $\vec{\jmath}$ is a unit vector … $\vec{\jmath} = \langle 0,1 \rangle$.



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a Vector in two forms …

A vector in component form … $\langle 8, 26 \rangle$

… can also be written as …

… the sum of unit vectors … $8\vec{i} + 26\vec{j}$

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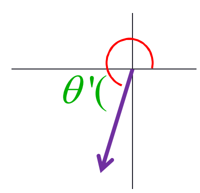
Example 1 … initial point: (-1, 5)
terminal point: (-2, -3)

a) Find component form.

$$\langle -2-(-1), -3-5 \rangle = \boxed{\langle -1, -8 \rangle}$$



b) Write as a sum of unit vectors.

$$\boxed{-\vec{i} - 8\vec{j}}$$

c) Find the magnitude.

$$\sqrt{1+64} = \sqrt{65} = \boxed{8.06}$$

d) Find the direction. Use [0°, 360°).

$$\theta' = \tan^{-1}\left(\frac{-8}{-1}\right) = \tan^{-1} 8 = 82.87°$$

$$\theta = 180° + 82.87° = \boxed{262.87°}$$

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Example 2 … Vector Operations

- Given $\vec{v} = 3\vec{\imath} - \vec{\jmath}$ and $\vec{w} = -2\vec{\imath} + 3\vec{\jmath}$.
- Find: $\vec{v} = \langle 3, -1 \rangle$ $\vec{w} = \langle -2, 3 \rangle$
  - a) $4\vec{v} + 2\vec{w} = 4\langle 3, -1 \rangle + 2\langle -2, 3 \rangle$
    $= \langle 12, -4 \rangle + \langle -4, 6 \rangle = \boxed{\langle 8, 2 \rangle}$ or $\boxed{8\vec{i} + 2\vec{j}}$
  - b) $\vec{v} - 3\vec{w} = \langle 3, -1 \rangle - 3\langle -2, 3 \rangle$
    $= \langle 3, -1 \rangle + \langle 6, -9 \rangle = \boxed{\langle 9, -10 \rangle}$ or $\boxed{9\vec{i} - 10\vec{j}}$
  - c) $\frac{1}{2}\vec{v} + \frac{1}{2}\vec{w} = \frac{1}{2}\langle 3, -1 \rangle + \frac{1}{2}\langle -2, 3 \rangle$
    $= \left\langle \frac{3}{2}, -\frac{1}{2} \right\rangle + \left\langle -\frac{2}{2}, \frac{3}{2} \right\rangle = \left\langle \frac{1}{2}, \frac{2}{2} \right\rangle = \boxed{\left\langle \frac{1}{2}, 1 \right\rangle} = \boxed{\frac{1}{2}\vec{i} + \vec{j}}$

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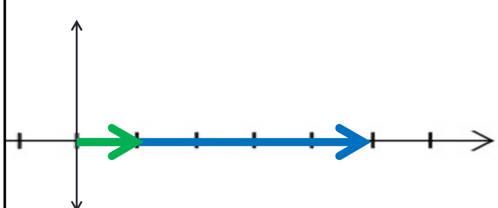
Demo to help explain the new formula you are about to see.



- consider the vector $\langle 5, 0 \rangle$
- it has magnitude 5
- a ***unit vector*** in the same direction would have magnitude 1
- that would be vector $\langle 1, 0 \rangle$

$$\frac{\langle 5, 0 \rangle}{5} = \frac{1}{5}\langle 5, 0 \rangle = \langle 1, 0 \rangle$$

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a unit vector in the direction of $\vec{v}$ …

A unit vector, $\vec{u}$, in the direction of $\vec{v}$ …

… is given by: $\vec{u} = \dfrac{\vec{v}}{\left\|\vec{v}\right\|}$ ← original vector ← original magnitude

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Example 3 … Find a unit vector in the direction of each given vector.

a) $\vec{v} = \langle 3, -4 \rangle$

$$\left\|\vec{v}\right\| = \sqrt{9+16} = \sqrt{25} = 5$$

$$\vec{u} = \frac{\langle 3, -4 \rangle}{5}$$

$$\vec{u} = \frac{1}{5}\langle 3, -4 \rangle$$

$$\vec{u} = \boxed{\left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle}$$

b) $-6\vec{i} + 4\vec{j}$

$$\left\|\vec{v}\right\| = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$\vec{u} = \frac{\langle -6, 4 \rangle}{2\sqrt{13}} = \frac{1}{2\sqrt{13}}\langle -6, 4 \rangle$$

$$= \left\langle -\frac{6}{2\sqrt{13}}, \frac{4}{2\sqrt{13}} \right\rangle = \left\langle -\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$$

$$\vec{u} = \boxed{\left\langle -\frac{3\sqrt{13}}{13}, \frac{2\sqrt{13}}{13} \right\rangle}$$

$$\text{or } \boxed{-\frac{3\sqrt{13}}{13}\vec{i} + \frac{2\sqrt{13}}{13}\vec{j}}$$

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