VECTOR APPLICATIONS
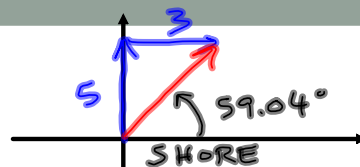
*** Use vectors in trig form!!! ***
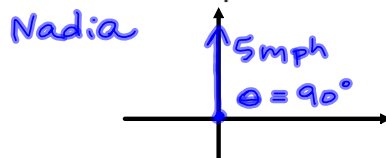
$$\vec{r} = \vec{a} + \vec{b}$$

$\|\vec{r}\| = $ resultant speed or resultant distance

Example 1

3

5 59.04°

SHORE

• Nadia is rowing across a river due north at a speed of 5 miles per hour perpendicular to the shore. The river has a current of 3 miles per hour heading due east.

Nadia

5mph
θ = 90°

current

θ = 0°
3mph

• a) At what speed is she heading?

$$\vec{r} = 5\langle\cos 90°, \sin 90°\rangle + 3\langle\cos 0°, \sin 0°\rangle$$
$$\vec{r} = \langle 5\cos 90° + 3\cos 0°, 5\sin 90° + 3\sin 0°\rangle$$
$$\vec{r} = \langle 3, 5\rangle \qquad \|\vec{r}\| = 5.83 \text{ mph}$$
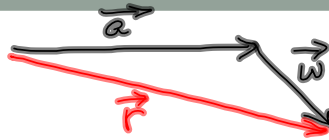
• b) What is her bearing with respect to the shore?

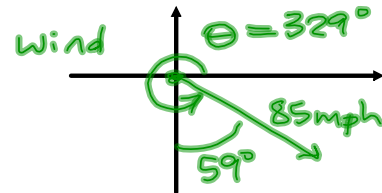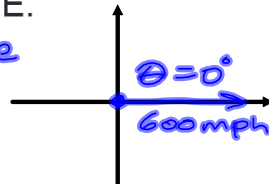$$\theta = \tan^{-1}\left(\frac{5}{3}\right) = 59.04°$$

$$\boxed{E \ 59.04° \ N}$$

Example 2

- An airplane is traveling due east with a speed of 600 miles per hour. The wind blows at 85 miles per hour at an angle of S 59° E.

airplane
$\theta = 0°$
600 mph

wind
$\theta = 329°$
85 mph
59°

- a) What is the resulting speed of the airplane?

$\vec{r} = 600\langle \cos 0°, \sin 0° \rangle + 85\langle \cos 329°, \sin 329° \rangle$

$\vec{r} = \langle 600\cos 0° + 85\cos 329°, 600\sin 0° + 85\sin 329° \rangle$
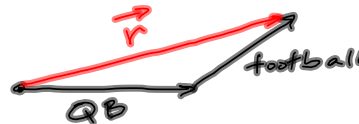
$\vec{r} = \langle 672.86, -43.78 \rangle$ $\|\vec{r}\| = 674.28$ mph

- b) What is the resulting bearing of the plane?

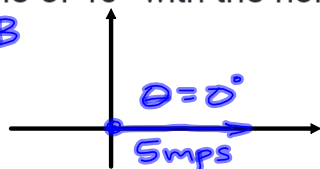$\theta = \tan^{-1}\left(\dfrac{-43.78}{672.86}\right) = 3.72°$ E 3.72° S
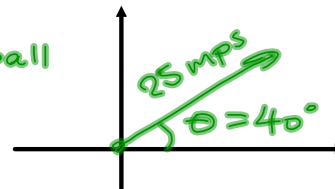

Example 3

- A quarterback running forward at 5 meters per second throws a football with a velocity of 25 meters per second at an angle of 40° with the horizontal.

QB
$\theta = 0°$
5 mps

football
25 mps
$\theta = 40°$

- a) What is the resultant speed of the pass?

$\vec{r} = 5\langle \cos 0°, \sin 0° \rangle + 25\langle \cos 40°, \sin 40° \rangle$

$\vec{r} = \langle 24.15, 16.07 \rangle$ $\|\vec{r}\| = 29.01$ mps

- b) What is the resultant bearing of the pass?

$\theta = \tan^{-1}\left(\dfrac{16.07}{24.15}\right) = 33.64°$ E 33.64° N

Example 4
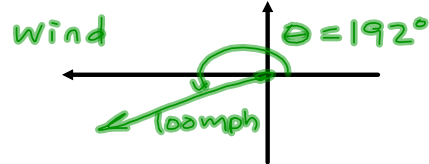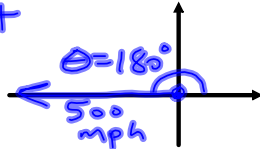
$\vec{r} = \vec{p} + \vec{w}$
$\vec{p} = \vec{r} - \vec{w}$

- A pilot needs to plot a course that will result in a velocity of 500 miles per hour in a direction of due west. If the wind is blowing 100 miles per hour from the directed angle of 192°, find the direction and speed the pilot should set to achieve this resultant.

resultant $\theta = 180°$ 500 mph

wind $\theta = 192°$ 100mph

$$\vec{p} = 500\langle \cos 180°, \sin 180° \rangle - 100\langle \cos 192°, \sin 192° \rangle$$

$$\vec{p} = \langle -402.19, 20.79 \rangle \qquad \|\vec{p}\| = 402.73 \text{ mph}$$

$$\theta' = \tan^{-1}\left(\frac{20.79}{-402.19}\right) = -2.96° \qquad \boxed{W \ 2.96° N}$$