Verifying Identities WS 1

1. $\cos^2 A \csc A \sec A = \cos^2 A \cdot \frac{1}{\sin A} \cdot \frac{1}{\cos A} = \frac{\cos A}{\sin A} = \cot A$

2. $\tan\beta(\sin\beta + \cot\beta\cos\beta) = \tan\beta\left(\sin\beta + \frac{\cos\beta}{\sin\beta}\cdot\cos\beta\right)$

$= \frac{\sin\beta}{\cos\beta}\left(\frac{\sin^2\beta + \cos^2\beta}{\sin\beta}\right) = \frac{1}{\cos\beta} = \sec\beta$

3. $\cos x(\sec x + \cos x \cdot \csc^2 x) = 1 + \cos^2 x \cdot \csc^2 x$

$= 1 + \frac{\cos^2 x}{\sin^2 x} = 1 + \cot^2 x = \csc^2 x$

4. $(\cos x - \sin x)^2 = \cos^2 x - 2\cos x\sin x + \sin^2 x$

$= 1 - 2\cos x\sin x$

5. $(\tan\beta + \cot\beta)^2 = \tan^2\beta + 2\tan\beta\cot\beta + \cot^2\beta$

$= (\tan^2\beta + 1) + (1 + \cot^2\beta) = \sec^2\beta + \csc^2\beta$

6. $\frac{1+\cot^2 x}{\sec^2 x} = \frac{\csc^2 x}{\sec^2 x} = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$

7. $\frac{\sec A}{\sin A} - \frac{\sin A}{\cos A} = \frac{1-\sin^2 A}{\sin A\cos A} = \frac{\cos^2 A}{\sin A\cos A} = \frac{\cos A}{\sin A} = \cot A$

8. $\frac{1}{1-\cos y} + \frac{1}{1+\cos y} = \frac{1+\cos y + 1 - \cos y}{1-\cos^2 y} = \frac{2}{\sin^2 y} = 2\csc^2 y$

9. $\cot^2 x \csc^2 x - \cot^2 x = \cot^2 x(\csc^2 x - 1) = \cot^4 x$

10. $\sec^4 a - \tan^4 a = (\sec^2 a + \tan^2 a)\cancel{(\sec^2 a - \tan^2 a)}$
$= 1 + \tan^2 a + \tan^2 a = 1 + 2\tan^2 a$

11. $\dfrac{1}{\sin x \cos x} - \dfrac{\cos x}{\sin x} \quad \dfrac{1-\cos^2 x}{\sin x \cos x} = \dfrac{\sin^2 x}{\sin x \cos x} = \dfrac{\sin x}{\cos x} = \tan x$

12. $\dfrac{1}{(1-\sin r)} \cdot \dfrac{(1+\sin r)}{(1+\sin r)} = \dfrac{1+\sin r}{1-\sin^2 r} = \dfrac{1+\sin r}{\cos^2 r}$
$= \dfrac{1}{\cos^2 r} + \dfrac{\sin r}{\cos^2 r} = \sec^2 r + \sec r \tan r$

13. $\dfrac{\cos x}{\sec x - 1} - \dfrac{\cos x}{\tan^2 x} = \dfrac{\cos x(\sec x + 1)}{\tan^2 x} - \dfrac{\cos x}{\tan^2 x}$
$= \dfrac{1 + \cos x - \cos x}{\tan^2 x} = \dfrac{1}{\tan^2 x} = \csc^2 x$

14. $\dfrac{\sec x}{\sec x - \tan x} = \dfrac{\sec x(\sec x + \tan x)}{\sec^2 x - \tan^2 x} = \sec^2 x + \sec x \tan x$

15. $\dfrac{(1+\sin x)(1+\sin x)}{(1-\sin^2 x)} = \dfrac{1 + 2\sin x + \sin^2 x}{\cos^2 x}$
$= \sec^2 x + 2\tan x \sec x + \tan^2 x$
$= \sec^2 x + 2\tan x \sec x + \sec^2 x - 1$
$= 2\sec^2 x + 2\tan x \sec x - 1$

16. $\sin^3 y - \sin^5 y = \sin^3 y\,(1-\sin^2 y)$

$= \sin^3 y \cos^2 y$

17. $\sec^2\theta + \csc^2\theta = \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$

$= \dfrac{1}{\sin^2\theta\cos^2\theta} = \csc^2\theta\sec^2\theta$

16. $\dfrac{1}{\sec\theta - \tan\theta} = \dfrac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta} = \sec\theta + \tan\theta$

19. $\dfrac{(1-4\cos x)}{(1-\cos x)} \cdot \dfrac{(1+\cos x)}{(1+\cos x)} = \dfrac{1-3\cos x - 4\cos^2 x}{1-\cos^2 x} = \dfrac{1-3\cos x - 4\cos}{\sin^2 x}$

20. $\dfrac{(\tan x - 4)}{(\tan x + 2)} \cdot \dfrac{\tan^2 x - 6\tan x - 8}{\tan^2 x - 4} = \dfrac{(\sec^2 x - 1) - 6\tan x - 8}{(\sec^2 x - 1) - 4}$

$= \dfrac{\sec^2 x - 6\tan x + 7}{\sec^2 x - 5}$

21. $\dfrac{\sec^3 y - \cos^3 y}{\sec y - \cos y} = (\cancel{\sec y - \cos y})(\sec^2 y + \sec y\cos y + \cos^2 y)$

$= \sec^2 y + 1 + \cos^2 y$

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

22. $(2\sin x + 3\cos x)^2 + (3\sin x - 2\cos x)^2$

$= 4\sin^2 x + 12\sin x\cos x + 9\cos^2 x$

$+ 9\sin^2 x - 12\sin x\cos x + 4\cos^2 x$

$= 13\sin^2 x + 13\cos^2 x = 13(\sin^2 x + \cos^2 x) = 13$

23. $\dfrac{(1+\sin x + \cos x)}{(1+\sin x - \cos x)} \cdot \dfrac{(1+\cos x)}{(1+\cos x)} = \dfrac{(1+\sin x+\cos x)(1+\cos x)}{1+\sin x - \cancel{\cos x} + \cancel{\cos x} + \sin x\cos x - \cos^2 x}$

$= \dfrac{(\quad)(\quad)}{\sin^2 x + \sin x + \sin x\cos x} = \dfrac{(\quad)(1+\cos x)}{\sin x(\quad)} = \dfrac{1+\cos x}{\sin x}$

24. $\dfrac{(1+\sin x + \cos x)}{(1-\sin x + \cos x)} \cdot \dfrac{(1+\sin x)}{(1+\sin x)} =$