

# Verifying Trigonometric Identities

Guidelines for  
Verifying Trig  
Identities

1. work with ONE side only
2. use algebraic technique  
(factor, add fractions, monomial denominators)
3. substitute using basic identities
4. convert everything to sin & cos
5. TRY SOMETHING!!

Example 1:

Verify  $\frac{\csc^2 x}{\cot x} = \csc x \sec x$

Start:  $\frac{\csc^2 x}{\cot x}$

$= \csc x \cdot \frac{\csc x}{\cot x}$

$= \csc x \cdot \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}}$

$= \csc x \cdot \frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cos x}$

$= \csc x \cdot \frac{1}{\cos x} = \csc x \cdot \sec x$

recip. sub.

quotient sub.

Example 2:

Verify  $\frac{\tan \theta}{1 + \sec \theta} + \frac{1 + \sec \theta}{\tan \theta} = 2 \csc \theta$

$$\begin{aligned}
 &= \frac{\tan \theta}{(1 + \sec \theta)} \cdot \frac{(1 - \sec \theta)}{(1 - \sec \theta)} + \frac{(1 + \sec \theta)}{\tan \theta} \\
 &= \frac{\tan \theta (1 - \sec \theta)}{(1 - \sec^2 \theta)} + \frac{(1 + \sec \theta)}{\tan \theta} \\
 &= \frac{\cancel{\tan \theta} (1 - \sec \theta)}{-\cancel{\tan^2 \theta}} + \frac{(1 + \sec \theta)}{\tan \theta} \\
 &= \frac{-(1 - \sec \theta)}{\tan \theta} + \frac{(1 + \sec \theta)}{\tan \theta} = \frac{-1 + \sec \theta + 1 + \sec \theta}{\tan \theta} \\
 &= \frac{2 \sec \theta}{\tan \theta} = \frac{\frac{2}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} = \frac{2}{\cos \theta} \cdot \frac{\cancel{\cos \theta}}{\sin \theta} = \frac{2}{\sin \theta} = 2 \csc \theta \quad !
 \end{aligned}$$

$1 + \tan^2 \theta = \sec^2 \theta$   
 $1 - \sec^2 \theta = -\tan^2 \theta$

Example 3:

Verify  $\frac{\cot^3 t}{\csc t} = \cos t (\csc^2 t - 1)$

start:  $\frac{\cot^3 t}{\csc t}$

$$\begin{aligned}
 &= \frac{\cos^3 t}{\sin^3 t} \cdot \frac{1}{\frac{1}{\sin t}} \\
 &= \frac{\cos^3 t}{\sin^3 t} \cdot \frac{\sin t}{1} \\
 &= \frac{\cos^3 t}{\sin^2 t} = \cos t \cdot \frac{\cos^2 t}{\sin^2 t} \\
 &= \cos t \cdot \cot^2 t \\
 &= \cos t \cdot (\csc^2 t - 1)
 \end{aligned}$$

start:  $\cos t (\csc^2 t - 1)$

$$\begin{aligned}
 &= \cos t \cdot \cot^2 t \cdot \frac{\cot t}{\cot t} \\
 &= \cos t \cdot \frac{\cot^3 t}{\cot t} \\
 &= \cos t \cdot \frac{\cot^3 t}{\frac{\cos t}{\sin t}} \\
 &= \cancel{\cos t} \cdot \cot^3 t \cdot \frac{\sin t}{\cancel{\cos t}} \\
 &= \cot^3 t \cdot \frac{1}{\csc t} \\
 &= \frac{\cot^3 t}{\csc t}
 \end{aligned}$$