

Writing Polynomial Equations Investigation

Name Fusion

Part 1:

a. Find ALL the zeros of $f(x) = x^3 - 8x^2 + 5x + 14$ given that $x = 2$ is one zero.

$$\begin{array}{r|rrrrr} 2 & 1 & -8 & 5 & 14 & \\ & & 2 & -12 & -14 & \\ \hline & 1 & -6 & -7 & 0 & \end{array}$$

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$$\boxed{x=2, x=7, x=-1}$$

b. Find ALL the zeros of $f(x) = 2x^3 - 16x^2 + 10x + 28$ given that $x = -1$ is one zero.

$$\begin{array}{r|rrrrr} -1 & 2 & -16 & 10 & 28 & \\ & & -2 & 18 & -28 & \\ \hline & 2 & -18 & 28 & 0 & \end{array}$$

$$x^2 - 9x + 14 = 0$$

$$(x-7)(x-2) = 0$$

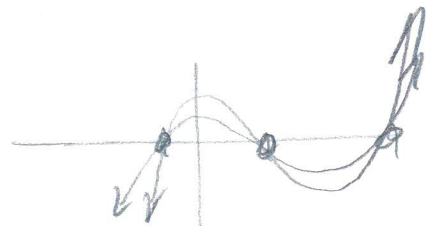
$$\boxed{x=-1, x=7, x=2}$$

c. What do you notice about the zeros of the two polynomial functions above? *Then are the same!*
 Are the functions the same? How are they different?

no one has a scalar of 2

d. Can more than one function have the exact same zeros?

yes!



Part 2:

To solve a polynomial function we first factor, and then set each factor equal to zero to get the roots/zeros. So, if we are given the zeros and want to write an equation, we work backwards!

a. The zeros of a quadratic function $q(x)$ are -3 and 7 . What are the factors?

$$(x+3)(x-7)$$

b. Write the quadratic function in standard form by multiplying the factors together. Don't forget " $q(x) =$ " ...

$$q(x) = x^2 - 4x - 21$$

c. This is not the only function that has those zeros! Can you write another one?

$$q_2(x) = 2x^2 - 8x - 42$$

$$q_3(x) = -x^2 + 4x + 21 \quad \text{etc.}$$

Part 3:

Extend what we did above, with quadratics, for polynomials of higher degree ... we just have more factors to multiply together! Write an equation in standard form for each function having the given zeros.

a. cubic function with zeros: 1, 2, -4

$$f(x) = (x-1)(x-2)(x+4)$$

$$f(x) = (x-1)(x^2+2x-8)$$

$$\underline{f(x) = x^3 + x^2 - 10x + 8}$$

b. quartic function with zeros: $\pm 2, \pm 3$

$$f(x) = (x-2)(x+2)(x-3)(x+3)$$

$$= (x^2-4)(x^2-9)$$

$$\underline{f(x) = x^4 - 13x^2 + 36}$$

c. cubic function with zeros: $-3, \pm i$... hint: multiply the two factors with imaginary numbers first!

$$f(x) = (x+3)(x-i)(x+i)$$

$$= (x+3)(x^2+1)$$

$$\underline{f(x) = x^3 + 3x^2 + x + 3}$$

d. cubic function with zeros: $2, \pm\sqrt{5}$... hint: multiply the two factors with irrational numbers first!

$$f(x) = (x-2)(x-\sqrt{5})(x+\sqrt{5})$$

$$= (x-2)(x^2-5)$$

$$\underline{f(x) = x^3 - 2x^2 - 5x + 10}$$

Part 4: ... Challenge yourself and be "the best of the best" ...

Write an equation in standard form for each function having the given zeros.

a. quadratic function with zeros: $2 \pm \sqrt{7}$

$$f(x) = x = 2 + \sqrt{7} \quad x = 2 - \sqrt{7}$$

$$f(x) = (x - 2 - \sqrt{7})(x - 2 + \sqrt{7})$$

$$\underline{f(x) = x^2 - 4x - 3}$$

b. quartic function with zeros: 1, 2, -4

★ repeat one zero / factor

$$f(x) = (x-1)^2(x-2)(x+4) \dots$$

OR

$$g(x) = (x-1)(x-2)^2(x+4) \dots$$

OR

$$h(x) = (x-1)(x-2)(x+4)^2 \dots$$