PreCalculus

Sum and Difference Identities WS 1 – Sine

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$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

Find the exact value of each expression.

1. $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$

$\sin\left(\frac{7\pi}{6}\right)\cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{7\pi}{6}\right)\sin\left(\frac{\pi}{3}\right)$

$\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$-\frac{1}{4} + \frac{3}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

2. $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3}$

$-\frac{1}{2} - \frac{\sqrt{3}}{2} = \boxed{\frac{-1-\sqrt{3}}{2}}$

Use the sum and difference formulas to find the exact values of the sine of the angle.

3. $75^\circ = 30^\circ + 45^\circ$

$\sin 75^\circ = \sin(30^\circ + 45^\circ)$

$= \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

4. $105^\circ = 60^\circ + 45^\circ$

$\sin 105^\circ = \sin(60^\circ + 45^\circ)$

$= \sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

5. $195^\circ = 225^\circ - 30^\circ$

$\sin 195^\circ = \sin(225^\circ - 30^\circ)$

$= \sin 225^\circ \cdot \cos 30^\circ - \cos 225^\circ \cdot \sin 30^\circ$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

6. $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$

$\sin\frac{11\pi}{12} = \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{3\pi}{4}\right) \cdot \cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

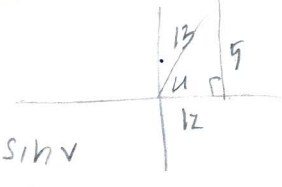
$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

Find the exact value of the trigonometric function given the following:

$\sin u = \frac{5}{13}, \quad 0 < u < \frac{\pi}{2}$ and $\cos v = -\frac{3}{5}, \quad \frac{\pi}{2} < v < \pi$

7. $\sin(u + v)$

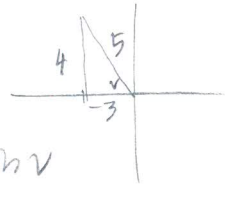


$= \sin u \cdot \cos v + \cos u \cdot \sin v$

$= \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{4}{5}\right)$

$= -\frac{15}{65} + \frac{48}{65} = \boxed{\frac{33}{65}}$

8. $\sin(u - v)$



$= \sin u \cdot \cos v - \cos u \cdot \sin v$

$= \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right) - \left(\frac{12}{13}\right)\left(\frac{4}{5}\right)$

$= -\frac{15}{65} - \frac{48}{65} = \boxed{-\frac{63}{65}}$